# The *AI Revolution* with 21$^{\text{st}}$ Century Skills: Implications for the Wage Inequality and Technical Change*

Rachael Grant[†] Murat Üngör[‡]

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**Abstract**

The future of work continues to come under speculation due to the growing presence of automation capital (such as industrial robots and artificial intelligence (AI)) in production. Previous and many of the current studies focus on how automation is replacing the tasks carried out specifically by unskilled labour. On top of this nexus, we focus on a new channel, AI and education (i.e., different types of high-skilled labour). We construct a three-level constant elasticity of substitution production model. In this model, labour is split into three components: (i) low-skilled labour, (ii) high-skilled labour with a traditional education background, and (iii) high-skilled labour with an AI-based education background. Several findings are presented. Firstly, our model confirms rising use of automation in production will cause a rise in the skill premium (wages of both types of high-skilled workers relative to low-skilled workers) and AI skill premium (wages of high-skilled labour with an AI-based education relative to high-skilled labour with a traditional education background). Secondly, our model demonstrates that perpetual long-run economic growth is possible despite there being no exogenous technological change. Lastly, our model demonstrates dependent on the value of the elasticity, automation will favour either high-skilled workers with a traditional education or high-skilled workers with an AI-based education background. This paper contributes to the literature by expanding on the focus placed on the elasticity of substitution between automation and unskilled/skilled workers by investigating the implication of AI and education on the future of work.

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[†]Enforcement and Resolution Department, Reserve Bank of New Zealand, 2 The Terrace, PO Box 2498, Wellington, New Zealand. E-mail address: Rachael.Grant@rbnz.govt.nz

[‡]Department of Economics, University of Otago, PO Box 56, Dunedin 9054, New Zealand. E-mail address: murat.ungor@otago.ac.nz

# 1 Introduction

Nobel laureate Herbert Simon famously said in 1965: "Machines will be capable, within twenty years, of doing any work a man can do."[1] Although his claim was premature for the time, in the current age of automation we are seeing the beginnings of machines substituting workers in some tasks at a potentially increasing rate. For example, a beef-boning machine being developed in New Zealand has "automating cutting equipment and scanning and vision technology" which is "more precise than manual labour."[2,3] Popular footwear brand Adidas has also built robot-only factories as robots start to become cheaper than labour.[4] These and many more examples tend to elicit fear that automation is a perfect substitute for human work and thus the future of work is in peril.

There are three main stylised facts presented in the related literature. The first fact is that wage inequality in the United States (U.S.) has seen an increasing trend since the 1970's.[5] Concurrent with this rise in wage inequality has been an increase in the skill premium; that is, the wages of workers with a college degree or higher have increased relative to wages of workers with a high school diploma or lower.[6] The second fact is that the increase in automation can help to explain the observed increase in the skill premium since the 1990's and thus helps to explain the overall rise in wage inequality in the U.S.[7] This is demonstrated by Figures 2, 6-8, and 10 which show an increase in robot installations over the past couple of years (excluding the small decrease in 2020 and 2021 due to COVID-19). The third fact is automation is likely to have a widespread effect on production, growth, and wage inequality as it is not limited to one industry.[8] This is demonstrated by Figures 2 and 10 which show some of the various industries where robot installations are occurring.

The above facts seem to suggest a picture where the future of work is in jeopardy but what if the very technology people fear will replace their jobs, could be the key to remaining in the labour force?

Education is a critical component of human capital especially when it comes to technological progress. Nelson and Phelps (1966) argue workers with higher education levels are better equipped to adapt, learn, and utilise new technology compared to workers with

[1] `https://www.ubs.com/microsites/nobel-perspectives/en/laureates/herbert-simon.html`

[2] This machine is being designed to carry out the task of beef boning due to the current labour shortage of workers in this area.

[3] `https://www.stuff.co.nz/business/farming/127376272/worlds-first-beefboning-machine-designed-in-dunedin`

[4] `https://www.zmescience.com/science/news-science/adidas-robot-factories-25052016`

[5] See, among others, Hornstein et al., 2005; Acemoglu and Autor, 2012; Autor, 2014; Lankisch et al., 2017.

[6] See, among others, Hornstein et al., 2005; Lankisch et al., 2017.

[7] See, among others, Katz and Murphy, 1992; Murphy and Welch, 1992; Juhn et al., 1993; Lankisch et al., 2017; Hémous and Olsen, 2022.

[8] See, among others, Frey and Osborne, 2017; Agrawal et al., 2019; Acemoglu and Restrepo, 2020.

lower education levels. Consequently, education becomes crucial to ensuring workers have the necessary skills to keep up with technological progress. Already there has been a shift in the education sector with the development of artificial intelligence (AI). Teachers are able to automate administrative tasks such as grading and AI tutors - based on machine learning - are being used to offer personalised learning for students. Perhaps the key to possessing skills to utilise automation technology is to learn from that technology.

The notion automation can help upskill human capital can be shown with the example of AlphaGo. AlphaGo, which was developed by DeepMind, is a computer programme designed to beat human Go players.[9,10] After the development of AlphaGo, Fan Hui, a three-time European Go Champion, was invited in 2015 to play against AlphaGo in London. He lost all five games. AlphaGo then played eighteen-time World Champion, Lee Sedol, and won four of the five games to the fear and amazement of everyone watching.[11] What is important about the AlphaGo example is the implication it can have for human capital. Fan Hui has come to see AlphaGo and AI as "a useful assistant, a tool to learn from" in order to get better at the game (Williams, 2019). AlphaGo also opened the Go community's eyes to new moves they had never seen before and moves considered 'bad' were shown to be pivotal moves for securing a win (Williams, 2019). This demonstrates how AI or automation, in general, can be used as tools to learn from and better one's skill set.

Motivated by the growing adoption of automation and the important role of education for human capital, we use an alternative lens regarding the future of work and automation. Previous and many of the current studies focus on how automation is replacing the tasks carried out specifically by unskilled labour (Acemoglu and Restrepo, 2019; Lankisch et al., 2019; Prettner, 2019). On top of this nexus, we focus on a new channel, AI and education (i.e., different types of high-skilled labour). This channel is worthy of investigation as AI in education could have implications for automation and the future of work.

To investigate the implication of AI in education, we construct a theoretical model which has five production factors: (i) traditional physical capital, (ii) automation capital, (iii) low-skilled labour, (iv) high-skilled labour with a traditional education background, and (v) high-skilled labour with an AI-based education background. The idea we develop is to split high-skilled labour into two categories. Here high-skilled workers with a traditional education background would be commonly known as high-skilled workers in the past literature. Using

[9] Go is one of the oldest board games in the world. The objective of the game is to claim more territory on the board than your opponent.

[10] All previous attempts of AI mastering the game Go failed and many believed that it would be a couple more decades before AI would achieve this goal. Even though AI has been able to master the game of chess, Go is much more complex (Williams, 2019).

[11] `https://www.youtube.com/watch?v=WXuK6gekU1Y`

the AlphaGo example, one could think of Fan Hui as belonging to the group of high-skilled workers with a traditional education background; however, after playing AlphaGo and using AlphaGo as a learning tool, Fan Hui was able to upskill himself in the game and would now belong to the group of high-skilled workers with an AI-based education background.

The theoretical model constructed is a three-level constant elasticity of substitution (CES) production model. Figure 1 illustrates this three-level CES model with Level 1 being the CES between the two types of high-skilled workers. Level 2 is the CES between high-skilled labour (this factor amalgamates both types of high-skilled labour) and low-skilled labour and automation capital. Note we assume low-skilled labour and automation capital are perfect substitutes, e.g., manufacturing workers are easily replaced by machinery. This assumption is not extended to high-skilled workers, e.g., managers are not easily replaced by automation. Lastly, Level 3 is the CES between the composite input (from Level 2) and traditional capital.

Our model yields several predictions. Firstly, we confirm the accumulation of automation will lead to a rise in the skill premium as wages of low-skilled workers fall while wages of both types of high-skilled workers rise (see Propositions 1-3). This is in line with conclusions reached by Lankisch et al. (2019). Secondly, the use of composite high-skilled labour in our model allows for the effects of automation between different types of high-skilled labour by introducing the notion of an AI skill premium. This is defined as the ratio between the wages of high-skilled workers with an AI-based education background and high-skilled workers with traditional education. This extends the literature which commonly focuses on the effect between unskilled and high-skilled labour only. Under our proposed model, we find that the AI skill premium falls when the relative supply of high-skilled workers with an AI-based education rises (see Proposition 4).

Next we construct a dynamical system to evaluate the impact of automation on long-run economic growth. Similar to the findings of Heer and Irmen (2019), Lankisch et al. (2019), and Prettner (2019), we characterise a long-run balanced growth path, where both types of capital in per capita terms and output per capita grow at the same rate, is possible with the addition of composite high-skilled labour to the model, even in the absence of technological progress (see Propositions 5-6). Lastly, we extend our analysis to illustrate how skill-biased technical change (SBTC) can be linked to the composite high-skilled labour and AI skill premium (see Propositions 7-8). However, whether automation favours high-skilled workers with an AI-based education or high-skilled workers with a traditional education background depends on the substitutability between the two types of workers (see Proposition 9).

This paper contributes to the literature in several ways. Firstly, we set forth a theoretical model which extends from the current literature in a novel approach, where we incorporate the impact of AI on human capital in a model of production via a labour input. Secondly, by incorporating AI-based education into the model, we are able to evaluate the implications of automation on wage inequality, economic growth, and SBTC adding to the insights of Heer and Irmen (2019), Lankisch et al. (2019), and Prettner (2019). Lastly, our findings explicitly show the crucial role education will play as automation technology becomes increasingly adopted by firms.

The remainder of this paper is structured as follows. Section 2 provides the motivation for our model and explains how we contribute to a gap in the literature. Section 3 introduces the model which is used to understand the implications of automation on wage inequality. A dynamical system is also constructed to understand the implications for economic growth. Section 4 then explores an extension to the model in regard to SBTC. Section 5 presents a discussion of plausible future research direction. Section 6 concludes. The Appendix presents the proofs and the technical details.

# 2 Motivation for Our Model

## 2.1 AI in Education: A Gap in the Literature

For centuries education has followed a homogenous approach to students by treating them as if they have the same level of intelligence (Milanesi, 2020). This has been an underlying problem with the educational system as it fails to account for students having different learning paces and styles (Milanesi, 2020). This problem is further exacerbated by the 'one teacher to many' structure which limits personalised learning for students (Milanesi, 2020). With technology continually developing, education tools incorporating technology are becoming more prominent in the education sector to try address this limitation.

One example of technology being used as an education tool is Tiro who is an educational assistant robot. The functions of Tiro include checking attendance, dancing, storytelling, role-playing, and providing conversation scripts (Hudson, 2019). Various educational applications (apps) have also been designed so students can continue to learn outside of the classroom.[12] However, there are limitations to these types of educational tools such as a lack of autonomy.[13] This is where AI technology can provide a solution.

With the development of AI, education tools can become autonomous. AI systems are

[12] These range from apps targeted at pre-school learning, all the way through to secondary schooling. Some examples include Kahn Academy, Quizlet, Prodigy, and Hopscotch (which teaches children how to code), among many more.

[13] This minimises the role robots can take as education tools as these robots need to be programmed to carry out tasks such as marking attendance. They cannot think for themselves.

being developed as educational tools such as online tutors which offer personalised learning for students. Personalised learning is achievable as AI systems learn from the student to gauge their learning style/pace and then teaches the student based on this.[14] Overall, this personalised learning can result in a better education (Milanesi, 2020).[15] AI can also be used to complete administrative tasks such as marking tests, further freeing up a teacher's time (Milanesi, 2020). With more time, teachers can switch to service-based tasks such as answering students' questions (Milanesi, 2020).[16] Overall, AI within education will result in a more efficient delivery of teaching and enable teachers to develop new teaching frameworks (AI Forum of New Zealand, 2018).

We focus on contributing to the gap in the literature regarding AI in education. Currently, the literature regarding AI and education has focused on the risks and ethical issues of AI whereas economic outcomes have remained relatively untouched in the research. If low-skilled workers are at most risk of being replaced, this will likely require upskilling of the labour force. Therefore, studying the impacts of AI on education is worth exploring.

## 2.2 The Inclusion of Composite High-Skilled Labour

The main difference between our model and previous models in the literature is the inclusion of composite high-skilled labour. As is standard practice in the literature, high-skilled labour is defined as requiring completion of a post-secondary schooling education, i.e., a university degree or equivalent. The composite high-skilled labour is then further differentiated as either high-skilled labour with a traditional education background or high-skilled labour with an AI-based education background. The motivation for this is due to the growing presence and development of AI-based education tools which will impact future levels of human capital.[17]

Nelson and Phelps (1966) argue the key role of human capital is to facilitate workers' ability to cope with change and technological progress rather than increasing productivity. Through education, workers can acquire the needed skills to effectively perform certain tasks

[14]Examples of this include Carnegie Learning who use AI as a part of their smart tutoring system and Smart Sparrow uses adaptive learning AI, which provides customised learning. See `https://www.smartsparrow.com/what-is-adaptive-learning/`

[15]This is because students would be less likely to fall behind as they are learning at their pace rather than the teacher moving things along too fast. Accelerated kids would also be learning at their faster pace rather than being held back because the teacher is going too slow.

[16]Another example of AI in education is Amy (`https://www.amy.app/`), which is an AI tutor for high school students created by the tutoring company; Jaipuna. Amy offers one-on-one tutoring and can give real time feedback on the student's progress. She can also learn from the data on her students to improve her overall teaching abilities.

[17]The AI Forum of New Zealand (2018) notes the education sector is one of the most labour-intensive sectors within NZ, therefore, the adoption of AI tools into this sector will "greatly improve the productivity of education" (AI Forum of New Zealand, 2018, p. 46).

or jobs. This is because education enables workers to better adapt to technological change as information can be better received, decoded, and understood by someone with a higher level of education (Nelson and Phelps, 1966). Economic growth then occurs as the ability for workers to innovate and adopt new technology rises as the level of human capital rises. Thus, Nelson and Phelps (1966) find a stronger association between levels of human capital and economic growth compared to the correlation between changes in human capital and economic growth. If the education sector adopts AI-based learning tools, the level of human capital will change and consequently so too will economic growth.

Supporting Nelson and Phelps' (1966) hypothesis, Foster and Rosenzweig (1995) empirically show that better educated farmers were more likely to adopt new technologies. Bartel and Lichtenberg (1987) find evidence of better educated workers having a comparative advantage in employing new technologies. Tang et al. (2021) also empirically find a strong association between education levels and skill levels, i.e., highly educated workers were strongly correlated as being high-skilled workers. Consequently, Tang et al. (2021) find firms adopting robots increase their demand for high-skilled/educated workers as they are better equipped to manage the robots. If AI-based education results in a higher level of human capital, then high-skilled workers will be better able to adapt to the growing adoption of automation technology in production compared to high-skilled workers with a traditional-based education.[18]

Duggan (2020) supports this reasoning by noting there has been a shift towards the "21$^{\text{st}}$ century skills" due to the growing presence of technology. These include, among some others, analytical and digital skills and experience in the use of technology. Through personalised learning provided by AI education tools, students will have the ability to achieve their full potential while developing these "21$^{\text{st}}$ century skills." This is consistent with evidence provided by Cassell et al. (2016) who note AI and multimodal social computing help to effectively develop cognitive, social, and emotional skills.[19]

Differentiation within high-skilled workers in the labour market, i.e., skilled labour from a traditional education background and skilled labour from an AI-based education background

[18]Traditional education is education we know today such as one teacher to many students. AI-based education differs by providing AI based education tools which can offer more personalised learning for each student.

[19]In the New Zealand context, recent reports have also emphasised the importance education will play in regard to AI and automation adoption. Due to the unpredictability of future innovation, Gavaghan et al. (2021) highlight the importance of producing graduates who have technological skills. The New Zealand Digital Skills Forum (2018) also argues AI will greatly help with the development of human capital and recommend exposure to technology be implemented in schools so students may understand the importance of possessing a digital skill set. It is therefore arguable that one would expect an AI-based education to result in a higher level of human capital compared to the level of human capital with a traditional education background. This is consistent with the Nelson-Phelps (1966) approach.

is plausible. One innovative example which shows this is an AI-based editing tool created by American Journal Experts (AJE) for academics.[20] This tool is able to edit papers in under ten minutes and is 95% more accurate than Grammarly (another AI-based editing tool) as it has used deep learning technology with AJE's own U.S.-trained editors.[21] The aim is to greatly reduce the number of hours academics use to manually edit their work so that they can accelerate the publication and sharing of other work. This example shows how AI-based tools can result in a higher level of human capital within the academic sector. For example, researchers who use the AJE AI-based editing tool can quickly publish better quality papers compared to those who manually edit their papers for publication. Thus, we can see how skilled labour can be differentiated by the use of AI-based tools.[22]

The model we propose includes low-skilled labour and two types of high-skilled labour: high-skilled labour from a traditional education background and high-skilled labour from an AI-based education background. Having three types of labour will allow for analysis into the effect of automation on wage inequality. Jaumotte et al. (2008) note that technological change often results in the wages of workers with certain skills increasing faster than workers with other skills which increases wage inequality. This is also supported by the Nelson-Phelps (1966) view of human capital which sees an increase in the earnings of skilled workers due to their ability to facilitate the adoption of new technology. Following these arguments, if AI-based educated workers are better equipped with the "21st century skills", then we would expect the proposed model to show a rise in wage inequality between those workers and the other two types of workers in the model. This argument is in line with the economic theory that technological progress increases wage inequality between skilled and unskilled labour as wages of skilled labour increase faster than wages of unskilled labour (Acemoglu, 2002b; Galor and Moav, 2000).

# 3 Model

## 3.1 Production

The proposed model is a nested CES model which considers a competitive economy in continuous time, $t \in [0, \infty]$, with population growth at rate $n$. The production of the final good, $Y(t)$, requires five inputs: (i) traditional capital, $K(t)$, (ii) automation capital (i.e.,

[20] `https://company.researchsquare.com/post/aje-launches-powerful-new-ai-based-editing-service?utm_source=sfmc&utm_medium=email&utm_campaign=IP+Warming+-+RS+Scholarly+Square+-+November+2021`

[21] These editors hold advanced degrees across hundreds of different academic areas.

[22] Another example which could lead to this differentiation is where where the younger generation (who have been brought up with AI-based education) enter the workforce with the older generation (who have been brought up with traditional based education).

industrial robots),[23] $P(t)$, (iii) low-skilled workers, $L_u(t)$, (iv) high-skilled workers with traditional education background, $L_s^T(t)$, and (v) high-skilled workers with AI-based education background, $L_s^P(t)$. The aggregate production function takes the form

$$Y(t) = A(t)\Big[a^{\frac{1}{\sigma_1}} K(t)^{\frac{\sigma_1 - 1}{\sigma_1}} + (1-a)^{\frac{1}{\sigma_1}} V(t)^{\frac{\sigma_1 - 1}{\sigma_1}}\Big]^{\frac{\sigma_1}{\sigma_1 - 1}}. \tag{1}$$

Here $A(t)$ is a measure of the current level of technical knowledge. Thus, improvements in the technological knowhow of the economy are captured by changes in $A(t)$. We abstract from exogenous technological change as done by Heer and Irmen (2019), Lankisch et al. (2019), and Prettner (2019), and set $A(t) = 1$ for all $t$. This allows us to study automation as the potential for perpetual economic growth. The parameter $a \in (0, 1)$ is a distribution parameter, which determines how important traditional capital, $K(t)$, and the composite input, $V(t)$, are in determining the production of the final good; and $\sigma_1 \in [0, \infty]$ is the elasticity of substitution between traditional capital and the composite input, $V(t)$. The composite input takes the form

$$V(t) = \Big[b^{\frac{1}{\sigma_2}} L_s(t)^{\frac{\sigma_2 - 1}{\sigma_2}} + (1-b)^{\frac{1}{\sigma_2}} [L_u(t) + P(t)]^{\frac{\sigma_2 - 1}{\sigma_2}}\Big]^{\frac{\sigma_2}{\sigma_2 - 1}}. \tag{2}$$

The parameter $b \in (0, 1)$ is a distribution parameter, high-skilled labour is denoted by $L_s(t)$, low-skilled labour is denoted by $L_u(t)$, and $P(t)$ denotes the automation capital (i.e., industrial robots). The parameter $\sigma_2 \in [0, \infty]$ is the elasticity of substitution. We follow Lankisch et al. (2019) and assume that $L_u(t)$ and $P(t)$ are perfect substitutes.[24] However, $L_s(t)$ and $P(t)$ are imperfect substitutes (e.g., it is hard for industrial robots/automation capital to replace managers). The composite high-skilled labour input takes the form

$$L_s(t) = \Big[c^{\frac{1}{\sigma_3}} L_s^T(t)^{\frac{\sigma_3 - 1}{\sigma_3}} + (1-c)^{\frac{1}{\sigma_3}} L_s^P(t)^{\frac{\sigma_3 - 1}{\sigma_3}}\Big]^{\frac{\sigma_3}{\sigma_3 - 1}}. \tag{3}$$

The parameter $c \in (0, 1)$ is a distribution parameter, which determines how important AI-based high-skilled labour, $L_s^P(t)$, and traditional-based high-skilled labour, $L_s^T(t)$, are in determining high-skilled labour $L_s(t)$. The parameter $\sigma_3 \in [0, \infty]$ is the elasticity of substitution between $L_s^P(t)$ and $L_s^T(t)$. As stated under the motivation for this model, these two types of high-skilled labour will likely differ in their skill set as one possesses more robust "21$^{\text{st}}$ century skills" compared to skilled labour with a traditional education background.

[23] Automation capital and industrial robots will be used interchangeably in this section. When industrial robots are mentioned, this is to highlight the perfect substitutability between low-skilled workers and industrial robots. For example, in the manufacturing sector or the automotive sector, industrial robots have replaced low-skilled workers in many parts of the production process.

[24] The defining property of perfect substitutes here is that the marginal rate of substitution between low-skilled labour and industrial robots is equal to unity.

Therefore, one would expect there to be a degree of substitutability between the two types of labour. For example, if a firm utilises an AI technology, they may substitute the high-skilled worker with a traditional education background for a high-skilled worker with an AI-based education background due to their deeper understanding of AI technology and how to use it.

Let $R_K(t)$, $R_P(t)$, $w_u(t)$, $w_s^T(t)$, and $w_s^P(t)$ denote the rental rate of traditional capital, the rental rate of industrial robots, the wage of low-skilled workers, the wage of high-skilled workers with a traditional education background, and the wage of high-skilled workers with an AI-based education background, respectively. The optimal plan of a competitive representative firm is to maximise profits

$$\Pi(t) = Y(t) - R_K(t)K(t) - R_P(t)P(t) - w_u(t)L_u(t) - w_s^T(t)L_s^T(t) - w_s^P(t)L_s^P(t). \tag{4}$$

All prices are expressed in units of contemporaneous output. We restrict our attention to the case where the representative firm's demand for all production factors is strictly positive for all $t$. The factor rewards are then given by

$$R_K(t) = \left(a\frac{Y(t)}{K(t)}\right)^{\frac{1}{\sigma_1}}. \tag{5a}$$

$$R_P(t) = \left((1-a)\frac{Y(t)}{V(t)}\right)^{\frac{1}{\sigma_1}} \left((1-b)\frac{V(t)}{L_u(t)+P(t)}\right)^{\frac{1}{\sigma_2}}. \tag{5b}$$

$$w_u(t) = \left((1-a)\frac{Y(t)}{V(t)}\right)^{\frac{1}{\sigma_1}} \left((1-b)\frac{V(t)}{L_u(t)+P(t)}\right)^{\frac{1}{\sigma_2}}. \tag{5c}$$

$$w_s^T(t) = \left((1-a)\frac{Y(t)}{V(t)}\right)^{\frac{1}{\sigma_1}} \left(b\frac{V(t)}{L_s(t)}\right)^{\frac{1}{\sigma_2}} \left(c\frac{L_s(t)}{L_s^T(t)}\right)^{\frac{1}{\sigma_3}}. \tag{5d}$$

$$w_s^P(t) = \left((1-a)\frac{Y(t)}{V(t)}\right)^{\frac{1}{\sigma_1}} \left(b\frac{V(t)}{L_s(t)}\right)^{\frac{1}{\sigma_2}} \left((1-c)\frac{L_s(t)}{L_s^P(t)}\right)^{\frac{1}{\sigma_3}}. \tag{5e}$$

## 3.2 Households

At all $t$ there are $L(t) > 0$ households. Each household is endowed with one unit of labour. This unit of labour is inelastically supplied to the labour market where $L(t) = L_u(t) + L_s^T(t) + L_s^P(t)$. The number of households grows at an exogenous instantaneous rate $n$ :

$\dot{L}(t) = nL(t)$.[25]

Following Heer and Irmen (2019), let $\mathcal{A}(t)$ denote household assets at $t$. The economy has two assets: the stock of traditional capital and the stock of industrial robots. Hence, $\mathcal{A}(t) = K(t) + P(t)$ for all $t$. Households are able to invest a portion of their income in either asset which will depend on the rate of return for each asset. The following no-arbitrage condition must be satisfied, so that we have an equilibrium with $K(t) > 0$ and $P(t) > 0$, as both assets are perfect substitutes as stores of value and depreciate at the same rate.

**Lemma 1.** *Since traditional and automation capital are perfect substitutes as stores of value, an equilibrium with $K(t) > 0$ and $P(t) > 0$ requires that households are indifferent between holding either type of capital. This suggests*

$$R_K(t) = R_P(t). \tag{6}$$

*Proof.* For low levels of traditional capital and for low levels of automation capital, Equation 5(a) and Equation 5(b) imply

$$\lim_{K(t)\to 0} R_K(t) = \infty. \tag{7a}$$

$$\lim_{P(t)\to 0} R_P(t) = \left((1-\alpha)\frac{Y(t)}{V(t)}\right)^{\frac{1}{\sigma_1}} \left((1-b)\frac{V(t)}{L_u(t)}\right)^{\frac{1}{\sigma_2}}. \tag{7b}$$

Equation 7(a) shows that the marginal product of traditional capital satisfies the Inada conditions.[26] Equation 7(b), however, shows that $\lim_{P(t)\to 0} R_P(t)$ is finite and that the Inada conditions are not fulfilled for automation capital such that the possibility of a corner solution emerges. Equations 7(a) and 7(b) indicate that if the traditional capital stock and the automation capital stock are close to zero, households will only want to invest in traditional capital, because its return is higher. Only later, for a large traditional capital stock, an interior equilibrium on the capital market emerges.[27] Such an interior equilibrium of the capital market is characterised by a no-arbitrage relationship between both types of investment implying $R_K(t) = R_P(t)$. ∎

**Lemma 2.** *Equilibrium factor prices for traditional capital, automation capital, and low-*

[25]For simplicity, we do not make a distinction between the population size and the amount of labour such that workforce growth and population growth coincide.

[26]In most macroeconomic and growth applications the Inada conditions (see Inada, 1963) ensure that solutions to the relevant dynamic optimisation problems lie in the interior of the feasible set. See Acemoglu (2009, Chapter 2) for illustrations.

[27]See also Heer and Irmen (2019) and Gasteiger and Prettner (2022) for related discussions.

*skilled workers coincide:*

$$R_K(t) = R_P(t) = w_u(t). \tag{8}$$

*Proof.* We know from Lemma 1 that equilibrium factor prices for traditional capital and automation capital are equal, i.e., $R_K(t) = R_P(t)$. Conditions (5b) and (5c) imply factor prices of automation capital and low-skilled labour must coincide as they are perfect substitutes and an equilibrium with $P(t) > 0$ and $L_u(t) > 0$ requires firms to be indifferent between the hiring of either factor. This suggests $R_P(t) = w_u(t)$. Combining this with Lemma 1 yields the above result. ∎

From national income accounting for a closed economy, the total amount of final goods in the economy must be either consumed or invested, thus

$$Y(t) = C(t) + I(t), \tag{9a}$$

where $C(t)$ is consumption and $I(t)$ is investment at time $t$. Under a closed economy (with no government spending), aggregate investment is equal to aggregate savings, $S(t)$:

$$S(t) = I(t) = Y(t) - C(t). \tag{9b}$$

As the proposed model is based on the Solow-Swan model, households save a constant portion of their income:

$$S(t) = sY(t), \tag{9c}$$

where $s \in (0, 1)$ is the exogenous saving rate implying households consume the remaining $(1 - s)$ portion of their income, and so

$$C(t) = (1 - s)Y(t). \tag{9d}$$

## 3.3 Equilibrium

Following Acemoglu (2009, Chapter 2) and Heer and Irmen (2019), the following definition for the equilibrium is given. Note households do not optimise when it comes to their savings or consumption decisions. Instead, their behaviour is captured by Equations (9c) and (9d). Nevertheless, firms still maximise profits and factor markets clear.

**Definition 1.** *Given initial values $L(0) > 0$, $K(0) > 0$ and $P(0) > 0$, and the evolution of population, $\dot{L}(t) = nL(t)$, an equilibrium path is a sequence*
$\{Y(t), K(t), P(t), C(t), S(t), R_K(t), R_P(t), w_u(t), w_s^T(t), w_s^P(t)\}_{t=0}^{\infty}$*, such that for all $t \geq 0$*
*(E1) The representative firm maximises profits.*

*(E2) Households are willing to hold both types of capital and save a constant fraction $s$ of their income.*
*(E3) The market for the final good clears, i.e., $C(t) + I(t) = Y(t)$.*
*(E4) The capital market clears, i.e., the demands for traditional and automation capital are equal to the respective supplies.*
*(E5) The labour market clears.*

### 3.4 Wage Levels, Wage Inequality, and Automation

The following propositions can be established with implications for the nexus between wages, wage inequality, and automation. Proposition 1 looks at the implication of a rise in the stock of automation capital on the wages of low-skilled workers while Proposition 2 studies this effect on the wages of both types of high-skilled workers. Proposition 3 shows the implication of rising automation capital on the skill premium between low and high-skilled workers. These first three propositions provide similar conclusions as Prettner (2019) and Heer and Irmen (2019) in regard to wage inequality and the skill premium. Proposition 4 provides a new insight into the skill premium by looking at the skill premium between different types of high-skilled labour.

To begin, consider the proposed nested CES model which implies automation and low-skilled labour are perfect substitutes, while automation and high-skilled labour are imperfect substitutes. In this context, we state the following results.

**Proposition 1.** *The accumulation of automation capital reduces the wages of low-skilled workers, i.e.,* $\frac{\partial w_u(t)}{\partial P(t)} < 0$.

*Proof.* See Appendix A.1.

As $\frac{\partial w_u(t)}{\partial P(t)} < 0$, the accumulation of industrial robots will decrease the wage rate of low-skilled workers. In contrast, the effect of an increase in the stock of industrial robots on the wages of high-skilled workers with a traditional education background and high-skilled workers with an AI-based education background is given by the following.

**Proposition 2.** *The accumulation of automation capital increases the wages of both types of high-skilled workers, i.e.,* $\frac{\partial w_s^T(t)}{\partial P(t)} > 0$ *and* $\frac{\partial w_s^P(t)}{\partial P(t)} > 0$.

*Proof.* See Appendix A.1.

As $\frac{\partial w_s^T(t)}{\partial P(t)} > 0$ and $\frac{\partial w_s^P(t)}{\partial P(t)} > 0$, the accumulation of industrial robots will increase the wage rate of high-skilled workers. Intuitively, if tasks performed by low-skilled workers are easily automated by industrial robots, then this would result in a fall in low-skilled workers' wages.

In contrast, both types of high-skilled labour see an increase in wages with the adoption of industrial robots.

**Proposition 3.** *The accumulation of automation capital leads to an increase in the skill premium, i.e.,* $\frac{\partial(w_s^T(t)/w_u(t))}{\partial P(t)} > 0$ *and* $\frac{\partial(w_s^P(t)/w_u(t))}{\partial P(t)} > 0$.

*Proof.* The skill premium is measured as the ratio between wages of high-skilled workers and low-skilled workers, which are given by the following wage ratios:

$$\frac{w_s^T(t)}{w_u(t)} = \left[\frac{b}{1-b}\frac{L_u(t)+P(t)}{L_s(t)}\right]^{\frac{1}{\sigma_2}}\left[c\frac{L_s(t)}{L_s^T(t)}\right]^{\frac{1}{\sigma_3}}, \tag{10a}$$

$$\frac{w_s^P(t)}{w_u(t)} = \left[\frac{b}{1-b}\frac{L_u(t)+P(t)}{L_s(t)}\right]^{\frac{1}{\sigma_2}}\left[(1-c)\frac{L_s(t)}{L_s^P(t)}\right]^{\frac{1}{\sigma_3}}. \tag{10b}$$

The effect of an increase in the stock of industrial robots on the skill premium is then

$$\frac{\partial(w_s^T(t)/w_u(t))}{\partial P(t)} = \frac{1}{\sigma_2}[L_u(t)+P(t)]^{\frac{1-\sigma_2}{\sigma_2}}\left[\frac{b}{1-b}\frac{1}{L_s(t)}\right]^{\frac{1}{\sigma_2}}\left[c\frac{L_s(t)}{L_s^T(t)}\right]^{\frac{1}{\sigma_3}}, \tag{11a}$$

$$\frac{\partial(w_s^P(t)/w_u(t))}{\partial P(t)} = \frac{1}{\sigma_2}[L_u(t)+P(t)]^{\frac{1-\sigma_2}{\sigma_2}}\left[\frac{b}{1-b}\frac{1}{L_s(t)}\right]^{\frac{1}{\sigma_2}}\left[(1-c)\frac{L_s(t)}{L_s^P(t)}\right]^{\frac{1}{\sigma_3}}. \tag{11b}$$

Given the specification of Equation (2), as long as $\sigma_2 > 0$, an increase in the stock of industrial robots increases the skill premium. ■

Intuitively, the rise in the skill premium is implied by Propositions 1 and 2. Low-skilled workers see a reduction in their wages while high-skilled workers see an increase in their wages with the adoption of industrial robots. This results in a rising skill premium.

Overall, the first three results are not surprising and align with the results discussed in the literature with different, albeit related models.[28] The fourth proposition we present below provides a new result to contribute to the literature.

**Proposition 4.** *An increase in relative supply of high-skilled workers with an AI-based education,* $L_s^P(t)/L_s^T(t)$, *lowers the relative wage of high-skilled workers with an AI-based education,* $w_s^P(t)/w_s^T(t)$.

*Proof.* Using Equations (5d) and (5e), we get the following AI skill premium

[28]See Prettner (2019) and Heer and Irmen (2019) for a detailed discussion on their similar results.

$$\frac{w_s^P(t)}{w_s^T(t)} = \left(\frac{1-c}{c}\frac{L_s^T(t)}{L_s^P(t)}\right)^{\frac{1}{\sigma_3}}. \tag{12a}$$

This can be written more conveniently in logarithmic form:

$$ln\left(\frac{w_s^P(t)}{w_s^T(t)}\right) = \frac{1}{\sigma_3}ln\left(\frac{1-c}{c}\right) - \frac{1}{\sigma_3}ln\left(\frac{L_s^P(t)}{L_s^T(t)}\right). \tag{12b}$$

Notice that

$$\frac{\partial ln\big(w_s^P(t)/w_s^T(t)\big)}{\partial ln\big(L_s^P(t)/L_s^T(t)\big)} = -\frac{1}{\sigma_3} < 0. \tag{12c}$$

This indicates the relative demand curve for the high-skilled workers with an AI-based education background versus high-skilled workers with a traditional education is downward sloping, given $\sigma_3 > 0$. An increase in relative supply, $L_s^P(t)/L_s^T(t)$, lowers the relative wage, $w_s^P(t)/w_s^T(t)$, with elasticity $\sigma_3$. ■

Intuitively, this is implied by economic theory. If there is a scarcity of high-skilled workers with an AI-based education but an abundance of high-skilled workers with a traditional education background, then assuming AI-based education provides better skills than traditional based education, one would expect the relative wage, $w_s^P(t)/w_s^T(t)$, to be high as firms increase the wage rate for AI educated workers. As the relative supply, $L_s^P(t)/L_s^T(t)$, increases due to the increase in demand by firms, the AI skill premium between the two types of high-skilled labour will fall in the short term, i.e., the relative wage, $w_s^P(t)/w_s^T(t)$, will fall. This is supported by Nelson and Phelps' technological gap concept where an increase in the technological gap should see an increase in the skill premium as demand for better educated workers increases.[29,30] Cummins and Violante (2002) find an empirical relationship between the technological gap and the skill premium in the U.S. by showing these have moved together over the past half century.[31] Acemoglu (1998) also reaches a similar conclusion. That is, technological advancement leads to an increase in the demand for skilled labour and demand for education which leads to more skilled labour. The increase in skilled labour then temporarily reduces the AI skill premium before a higher AI skill premium eventuates due to more technological progress.

[29] The technological gap is defined as the percentage difference between the technology operated by old machines and the new machines (Nelson and Phelps, 1966).

[30] This is because better educated workers are assumed to better adapt to the new technology (Nelson and Phelps, 1966).

[31] Cummins and Violante (2002) use U.S. data on the relative prices and quantities of equipment investment to find this relationship.

### 3.5 Dynamical System

Originally developed for car manufacturing, industrial robots have quickly dispersed throughout the manufacturing sector and into other sectors as well (see Figure 2). In the manufacturing sector alone, robots per worker has increased from approximately two robots per one thousand workers in 2001 to ten robots per one thousand workers in 2015 (Alonso et al., 2022).[32] Therefore, as robots, AI, and other automation capital continue to be adopted by firms, long-run economic growth will be impacted. Both Prettner (2019) and Lankisch et al. (2019) construct a dynamical system to analyse this long-run impact. Importantly, both find automation adoption can lead to perpetual economic growth in the absence of technological change. To further enrich our model, a similar dynamical process can be adapted here.

We describe the economy's evolution in terms of per capita variables, which are denoted by lowercase letters. For any variable $X(t)$, we have $x(t) \equiv X(t)/L(t)$, where $L(t) = L_u(t) + L_s^T(t) + L_s^P(t)$. The economy's evolution can then be described using the following per capita variables:

$$\begin{aligned}&\lambda(t) \equiv \frac{\mathcal{A}(t)}{L(t)}, k(t) \equiv \frac{K(t)}{L(t)}, p(t) \equiv \frac{P(t)}{L(t)}, l_s(t) \equiv \frac{L_s(t)}{L(t)}, l_u(t) \equiv \frac{L_u(t)}{L(t)}, l_s^P(t) \equiv \frac{L_s^P(t)}{L(t)},\\ &l_s^T(t) \equiv \frac{L_s^T(t)}{L(t)}, \qquad (13)\end{aligned}$$

where the above denote total assets, traditional capital, automation capital, high-skilled labour, low-skilled labour, high-skilled labour with an AI-based education, and high-skilled labour with a traditional education background in per capita terms, respectively. From this we derive output per capita, $y(t) \equiv \frac{Y(t)}{L(t)}$, as

$$y(t) = \left[a^{\frac{1}{\sigma_1}} k(t)^{\frac{\sigma_1 - 1}{\sigma_1}} + (1-a)^{\frac{1}{\sigma_1}} v(t)^{\frac{\sigma_1 - 1}{\sigma_1}}\right]^{\frac{\sigma_1}{\sigma_1 - 1}}, \qquad (14)$$

where $v(t) \equiv V(t)/L(t)$ denotes the composite input per capita.

**Lemma 3.** *The equilibrium stock of automation capital per capita depends on the automation capital per capita and all types of labour and it is given by:*

$$k(t) = v(t)\left(\frac{a}{1-a}\right)\left(\frac{1}{1-b}\right)^{\frac{\sigma_1}{\sigma_2}}\left(\frac{l_u(t) + p(t)}{v(t)}\right)^{\frac{\sigma_1}{\sigma_2}}, \qquad (15)$$

[32] This is based on high-income countries and the data on the number of robots comes from the International Federation of Robotics (IFR), the manufacturing sector employment data comes from various sources collected by Alonso et al. (2022) which can be found in their data appendix.

*where*

$$v(t) = \left[ b^{\frac{1}{\sigma_2}} \left[ c^{\frac{1}{\sigma_3}} l_s^T(t)^{\frac{\sigma_3-1}{\sigma_3}} + (1-c)^{\frac{1}{\sigma_3}} l_s^P(t)^{\frac{\sigma_3-1}{\sigma_3}} \right]^{\frac{\sigma_3}{\sigma_3-1}\frac{\sigma_2-1}{\sigma_2}} + (1-b)^{\frac{1}{\sigma_2}} [l_u(t) + p(t)]^{\frac{\sigma_2-1}{\sigma_2}} \right]^{\frac{\sigma_2}{\sigma_2-1}}.$$

*Proof.* See Appendix A.1.

Rearranging Equation (15) we can also write the equilibrium stock of automation capital per capita as

$$p(t) = \left(\frac{1-a}{a}\right)^{\frac{\sigma_2}{\sigma_1}} \left(\frac{k(t)}{v(t)}\right)^{\frac{\sigma_2}{\sigma_1}} (1-b)v(t) - l_u(t). \tag{16}$$

Recall that $\mathcal{A}(t) = K(t) + P(t)$ for all $t$. The equilibrium conditions (E2) and (E3) imply that $I(t) = sY(t)$, where the aggregate investment is used to raise the stocks of both types of capital. This leads to the following lemma.

**Lemma 4.** *The evolution of total assets per capita is governed by*

$$\dot{\lambda}(t) = sy(t) - (\delta + n)\, a\,(t). \tag{17}$$

*Proof.* See Appendix A.1.

From this dynamical system, the following propositions hold.

**Proposition 5.** *The asymptotic growth rate of traditional capital per capita, automation capital per capita, and output per capita are all equal:*

$$\frac{\dot{k}(t)}{k(t)} = \frac{\dot{p}(t)}{p(t)} = \frac{\dot{y}(t)}{y(t)}. \tag{18}$$

*Proof.* See Appendix A.1.

**Proposition 6.** *The economy exhibits a long-run balanced growth, $g$:*

$$g \equiv \frac{\dot{k}(t)}{k(t)} = \frac{\dot{p}(t)}{p(t)} = \frac{\dot{y}(t)}{y(t)} = \frac{sD_1 - (\delta+n)D_2}{D_2}. \tag{19}$$

*This growth rate is positive as long as $sD_1 - (\delta+n)D_2 > 0$, where $D_1$ and $D_2$ are defined as*

$$D_1 \equiv \left[ a(1-a)^{\frac{1-\sigma_1}{\sigma_1}} (1-b)^{\frac{\sigma_2-\sigma_1}{\sigma_1\sigma_2}} + (1-a)^{\frac{1}{\sigma_1}} (1-b)^{\frac{\sigma_1-1}{\sigma_1(\sigma_2-1)}} \right],\ D_2 \equiv 1 + \left(\tfrac{a}{1-a}\right)(1-b)^{\frac{1-\sigma_1}{\sigma_2-1}}.$$

*Proof.* See Appendix A.1.

From Proposition 6, one can see that even in the absence of exogenous technical change, the economy can grow at a constant rate, $g$, as long as $sD_1 - (\delta+n)D_2 > 0$.

# 4 Incorporating Skill-Biased Technical Change

## 4.1 Differentiation within High-Skilled Workers and SBTC

The dramatic rise in U.S. earnings inequality and the changes in the returns to college from the 1970's to today has led to a growing literature which attributes these changes to skilled-biased technical change (SBTC).[33,34] This is because the effects of technology on the labour market are a concern for economists due to the opportunities created by technological change for some workers while other opportunities disappear for others.

Technical change is often not neutral as it benefits some factors of production more than others. In recent decades, for example, there has been a shift towards technology complementing skilled labour, i.e., SBTC.[35] For this technical change to be skill-biased, the shift in the production function due to technological change has to favour skilled over unskilled labour (Violante, 2008). Violante (2008) notes there are different factors which make a worker more skilled than another and hence why technological change may favour skilled labour more. One of these factors is education.[36]

Recall that the composite high-skilled labour input takes the following form in Equation (3), $L_s(t) = \left[c^{\frac{1}{\sigma_3}} L_s^T(t)^{\frac{\sigma_3-1}{\sigma_3}} + (1-c)^{\frac{1}{\sigma_3}} L_s^P(t)^{\frac{\sigma_3-1}{\sigma_3}}\right]^{\frac{\sigma_3}{\sigma_3-1}}$. We now introduce the following skill-augmenting technology terms into Equation (3), $A_s^T(t)$ and $A_s^P(t)$.

$$L_s(t) = \left[c^{\frac{1}{\sigma_3}} \left(A_s^T(t)L_s^T(t)\right)^{\frac{\sigma_3-1}{\sigma_3}} + (1-c)^{\frac{1}{\sigma_3}} \left(A_s^P(t)L_s^P(t)\right)^{\frac{\sigma_3-1}{\sigma_3}}\right]^{\frac{\sigma_3}{\sigma_3-1}}. \tag{20}$$

In Equation (20), $A_s^T(t)$ and $A_s^P(t)$ are the so-called efficiency terms, which are functions of time alone. A rise in either is referred to as factor augmenting technical change. We define $A_s^T(t)L_s^T(t)$ and $A_s^P(t)L_s^P(t)$ as "effective traditional-based high-skilled labour" and "effective AI-based high-skilled labour," respectively. Technologies are factor augmenting in that they augment the productivity of high-skilled workers with a traditional education or high-skilled workers with an AI-based education background by raising $A_s^T(t)$ or $A_s^P(t)$.[37]

**Proposition 7.** *Changes in the skill-bias of technology, reflected in the evolution of* $\frac{A_s^P(t)}{A_s^T(t)}$,

[33] Acemoglu and Autor (2011) provide a comprehensive review.

[34] See, among many others, Katz and Murphy (1992), Murphy and Welch (1992), Juhn et al. (1993) for more on the rising U.S. earnings inequality and the rising returns to college since the 1970's.

[35] Berman et al. (1994) find computer purchases and R&D expenditures account for 70% of the shift away from production to non-production labour. Murphy and Welch (1992) find an increase in the share of college labour in all sectors since the late 1970's and an increase in the college premium.

[36] Other factors include innate ability and experience (Violante, 2008).

[37] Technological change is said to be purely traditional-based high-skilled labour augmenting if $\dot{A}_s^P(t) = 0$ and $\dot{A}_s^T(t) > 0$, whereas it is purely AI-based high-skilled labour augmenting if $\dot{A}_s^T(t) = 0$ and $\dot{A}_s^P(t) > 0$. It is equally traditional-based high-skilled labour and AI-based high-skilled labour augmenting if $\dot{A}_s^P(t) = 0$ and $\dot{A}_s^T(t) = 0$, where an "overdot" denotes time derivative.

*and changes in the relative supply of skills, reflected in the evolution of* $\frac{L_s^P(t)}{L_s^T(t)}$*, determine the AI skill premium,* $\frac{w_s^P(t)}{w_s^T(t)}$*. That is,* $ln\left(\frac{w_s^P(t)}{w_s^T(t)}\right) = \frac{1}{\sigma_3}ln\left(\frac{1-c}{c}\right) + \frac{\sigma_3-1}{\sigma_3}ln\left(\frac{A_s^P(t)}{A_s^T(t)}\right) - \frac{1}{\sigma_3}ln\left(\frac{L_s^P(t)}{L_s^T(t)}\right)$*.*

*Proof.* Recall the proof for Proposition 4. Using (5d) and (5e), we found the AI skill premium to take the form of Equation (12a): $\frac{w_s^P(t)}{w_s^T(t)} = \left(\frac{1-c}{c}\frac{L_s^T(t)}{L_s^P(t)}\right)^{\frac{1}{\sigma_3}}$. To account for the skill augmenting technology terms we have introduced into Equation (3), Equation (12a) can be rewritten as

$$\frac{w_s^P(t)}{w_s^T(t)} = \left(\frac{1-c}{c}\right)^{\frac{1}{\sigma_3}} \left(\frac{A_s^P(t)}{A_s^T(t)}\right)^{\frac{\sigma_3-1}{\sigma_3}} \left(\frac{L_s^P(t)}{L_s^T(t)}\right)^{-\frac{1}{\sigma_3}}. \tag{21a}$$

This can be written more conveniently in logarithmic form:

$$ln\left(\frac{w_s^P(t)}{w_s^T(t)}\right) = \underbrace{\frac{1}{\sigma_3}ln\left(\frac{1-c}{c}\right)}_{\text{Constant}} + \underbrace{\frac{\sigma_3-1}{\sigma_3}ln\left(\frac{A_s^P(t)}{A_s^T(t)}\right)}_{\substack{\text{Changes in the}\\ \text{skill-bias of technology}}} - \underbrace{\frac{1}{\sigma_3}ln\left(\frac{L_s^P(t)}{L_s^T(t)}\right)}_{\substack{\text{Changes in the}\\ \text{relative supply of skills}}} \tag{21b}$$

■

Equation (21b) shows the two key mechanisms to understand the changes in the AI skill premium. The second term on the right-hand side of Equation (21b) links the SBTC to the AI skill premium, whereas the last term on the right-hand side of Equation (21b) shows that the greater the relative number of high-skilled workers with an AI-based education, the lower their relative wage. For a given skill bias of technology, an increase in the relative supply of high-skilled workers with an AI-based education reduces the skill premium with an elasticity of $1/\sigma_3$. Accordingly, the factor which is important for both forces is the elasticity of substitution between these two types of high-skilled workers, $\sigma_3$. This is because how easily one type can substitute for the other will determine the influence of the SBTC and thus the skill premium. Proposition 8 and Proposition 9 show the importance of $\sigma_3$.

**Proposition 8.** *Given that* $\sigma_3 \geq 0$*, the relative demand curve for high-skilled workers with an AI-based education versus high-skilled workers with traditional education is downward sloping. That is,* $\frac{\partial ln(w_s^P(t)/w_s^T(t))}{\partial ln(L_s^P(t)/L_s^T(t))} = -\frac{1}{\sigma_3}$*.*

*Proof.* Equation (21b) relates the *relative* wage of high-skilled workers with an AI-based education, $ln(w_s^P(t)/w_s^T(t))$, to their *relative* quantity, $ln(L_s^P(t)/L_s^T(t))$. This gives the *relative demand curve* for high-skilled workers with an AI-based education. Differentiating Equation (21b) with respect to $L_s^P(t)/L_s^T(t)$ yields:

$$\frac{\partial ln(w_s^P(t)/w_s^T(t))}{\partial ln(L_s^P(t)/L_s^T(t))} = -\frac{1}{\sigma_3}. \tag{22}$$

■

The elasticity of substitution between the two types of high-skilled labour pins down the steepness of the demand curve.

Figure 3 illustrates the two key mechanisms of Equation (21b).[38] There are two time periods, denoted 0 and 1. The vertical axis shows the AI skill premium, $w_s^P/w_s^T$, and the horizontal axis represents the relative quantity of high-skilled workers with an AI-based education, $L_s^P/L_s^T$. The downward-sloping relative demand curve (see Proposition 8) implies that employers hire relatively few high-skilled workers with an AI-based education when their relative wage is high. The perfectly inelastic supply curve indicates that the relative number of high-skilled workers with an AI-based education is fixed. The labour market for high-skilled workers is in equilibrium at point $A$.

The *changes in the relative supply of skills* mechanism in Equation (21b) can be discussed as follows: a rise in the relative supply of high-skilled workers with an AI-based education would shift the relative supply curve to the right (from $S_0$ to $S_1$). This supply shift would move the labour market for high-skilled workers to point $C$, reducing the AI skill premium by the amount of $\Delta_S$. This is analog to the last term on the right-hand side of Equation (21b).

Assuming that $A_s^P(t) > A_s^T(t)$ and $\sigma_3 > 1$ (see Proposition 9 for this), the second term on the right-hand side of Equation (21b), $\frac{\sigma_3-1}{\sigma_3} ln\left(\frac{A_s^P(t)}{A_s^T(t)}\right)$, is positive and increases the AI skill premium. If the demand shift is sufficiently large (from $D_0$ to $D_1$), the new equilibrium at point $B$ is characterised by a larger wage gap between high-skilled workers with an AI-based education versus high-skilled workers with traditional education by the amount of $\Delta_D$.

Equations (21b) and Equation (22) imply that for given skill bias, $A_s^P(t)/A_s^T(t)$, an increase in relative supplies, $L_s^P(t)/L_s^T(t)$, lowers relative wages with an elasticity of $1/\sigma_3$, given that $\sigma_3 \geq 0$. Note that one can rearrange Equation (22) to derive the elasticity of substitution between the two types of high-skilled labour, $\sigma_3 = -\left(\frac{\partial ln(w_s^P(t)/w_s^T(t))}{\partial ln(L_s^P(t)/L_s^T(t))}\right)^{-1}$.

**Proposition 9.** *The relationship between the relative productivity of two types of high-skilled labour $A_s^P(t)/A_s^T(t)$, and the AI skill premium within the two types of the high-skilled workers, $w_s^P(t)/w_s^T(t)$, is purely determined by the elasticity of substitution between these two types of high-skilled labour. That is,* $\frac{\partial ln(w_s^P(t)/w_s^T(t))}{\partial ln(A_s^P(t)/A_s^T(t))} = \frac{\sigma_3-1}{\sigma_3}$.

*Proof.* The proof directly follows Equation (21b). Differentiating Equation (21b) with respect to $A_s^P(t)/A_s^T(t)$ yields:

$$\frac{\partial ln(w_s^P(t)/w_s^T(t))}{\partial ln(A_s^P(t)/A_s^T(t))} = \frac{\sigma_3 - 1}{\sigma_3}. \tag{23}$$

[38]We ignore the effect of the constant term, the first term on the right-hand side of Equation (21b), in this illustration. This term vanishes if we set c=0.5.

■

The larger $\sigma_3$ is, the greater the impact of changes in technology, $A_s^P(t)/A_s^T(t)$, on the AI skill premium relative to changes in relative skill supplies. As high-skilled workers with a traditional education and high-skilled workers with an AI-based education background become closer substitutes, the skill premium becomes more responsive to any SBTC-induced demand shifts.

Recall $\sigma_3 > 0$; therefore, the relationship between the relative productivity of the two types of high-skilled workers and the skill premium within high-skilled workers depends on the value of $\sigma_3$. This results in two cases: (i) $\sigma_3 > 1$, and (ii) $\sigma_3 < 1$. Thus, the elasticity of substitution between the two types of high-skilled workers, $\sigma_3$, is important as it will inform us which type of high-skilled worker is favoured by technical change.

For Case 1, where the elasticity of $\sigma_3 > 1$, high-skilled workers with a traditional education and high-skilled workers with an AI-based education background are *gross substitutes*. This means that a reduction in supply of the high-skilled workers with a traditional education creates added demand for the high-skilled workers with an AI-based education background. Therefore, an increase in the productivity of the high-skilled workers with an AI-based education background (i.e., $A_s^P(t)/A_s^T(t)$ increases) will lead to an increase in their wages (i.e., $w_s^P(t)/w_s^T(t)$ increases). This is depicted in Figure 4.

For Case 2, where the elasticity of $\sigma_3 < 1$, high-skilled workers with a traditional education and high-skilled workers with an AI-based education background are *gross complements*. That is, a *high-skilled worker with an AI-based education background-augmenting technological change* (or a rise in $A_s^P(t)$) actually increases demand for the complementary input (high-skilled workers with a traditional education background) more than it increases the demand for high-skilled workers with an AI-based education background. The excess demand for high-skilled workers with a traditional education background raises its marginal product more than that of high-skilled workers with an AI-based education background, leading to a *high-skilled worker with a traditional education background-bias* in production.[39] Figure 5 visualises this case with a decrease in the relative wage (i.e., $w_s^P(t)/w_s^T(t)$ decreases). A reduction in supply of the high-skilled workers with a traditional education reduces demand for the high-skilled workers with an AI-based education background.

[39]Similarly, *high-skilled workers with a traditional education background-augmenting technological change* (or a rise in $A_s^T(t)$) leads to a *high-skilled worker with an AI-based education background-bias* when $\sigma_3 < 1$.

## 4.2 The Two Elasticities of Interest

### 4.2.1 The Elasticity of Substitution Between Skilled and Unskilled Workers

The empirical literature has mainly focused on estimates of elasticity for skilled workers versus unskilled workers (Katz and Murphy, 1992; Berman et al., 1994; Machin and Van Reenen, 1998; Autor et al., 2008; Tang et al., 2021). This literature has tended to reach a consensus regarding the elasticity of substitution between skilled and unskilled workers ($\sigma$) by finding $\sigma > 1$ (Katz and Murphy, 1992; Acemoglu, 2002a; Ciccone and Peri, 2005). Studies using U.S. micro data tend to converge towards an estimate of 1.6 for the elasticity of substitution between skilled and unskilled workers (Jerzmanowski and Tamura, 2020).[40] However, when applied in an international dimension, Jerzmanowski and Tamura (2020) find this elasticity to fall between 1.8 and 2.6.[41] This is an important finding because even at an international level $\sigma > 1$.

Due to the literature's consistency in estimating the elasticity of substitution between unskilled and skilled labour, Havranek et al. (2021) review the current literature to assess the extent these estimations suffer from publication and attenuation bias.[42] They note the empirical estimates are likely to suffer from attenuation bias due to labour supply data being "notoriously noisy."[43] Thus, reported estimates are likely to be representative of the lower bound. However, most of the literature accounts for this bias (Katz and Murphy, 1992; Angrist, 1995; Borjas, 2003). The publication bias is found to stem from the underreporting of small estimates in the literature. This arises due to the intuition that the elasticity of substitution between skilled and unskilled labour is nonnegative. Consequently, small and negative values are often discriminated against resulting in this bias. After collecting 1096 estimates from 99 studies, Havranek et al. (2021) find correcting for attenuation and publication bias results in an estimate of 4 for the elasticity of substitution between skilled and unskilled workers. This is significantly higher than the common estimation of around 1.5 in the literature, suggesting that skilled and unskilled workers are more substitutable than what is believed. Therefore, this parameter estimate can be applied in our model to the elasticity between high-skilled and low-skilled workers in Equation (2), that is $\sigma_2 > 1$.

[40] Other empirical work has shown an elasticity of 1.4 and have used a wide range of data sets including time series and cross sections (Hornstein and Krusell, 2005). For example, see Hamermesh (1995).

[41] Jerzmanowski and Tamura mostly focus on developed countries. The elasticity of substitution value varies between these economies however, it still remains greater than one.

[42] Publication bias occurs because some empirical literature is more likely to be published than others. Attenuation bias is defined as bias in an estimator towards zero because the independent variable almost always suffers from a measurement error.

[43] This is due to measurement error from survey responses, migrant degrees not being comparable to native degrees due to different education systems, and noise can be created from mapping of degrees to skills (Havranek et al., 2021).

We also have the elasticity between automation capital and skilled workers, and automation capital and unskilled workers, $\sigma_2$, in Equation (2). The empirical literature also analyses this elasticity of substitution. For example, both Berg et al. (2018) and Alonso et al. (2022) use robot capital to model the AI revolution due to the high substitutability between robots and labour compared to previous technologies.[44] Alonso et al. (2022) analyse real wages and robot density in the manufacturing sector using data from the International Federation of Robotics (IFR).[45] They find a positive relationship (slope is greater than 1) between robot density and real wages over time which they argue is "plausibly in line with high elasticity of substitution between robots and labour" (Alonso et al., 2022, p. 6). That is $\sigma$ lies above one, so in our case $\sigma_2 > 1$. Further, this positive relationship is likely indicative of the lower bound due to robots being a subset of automation.

IFR data supports this hypothesis; $\sigma_2 > 1$. Figure 6 shows that annual robot installations have increased from 2010 albeit with a slight decrease in 2020 due to the COVID-19 pandemic. Further installations, however, are projected to continue increasing up to 2024 which can be seen in Figure 7. Robot density in the manufacturing sector by country is also shown in Figure 8 with France having a robot density of 194 robots per 10,000 workers in 2020. This is an increase from 2016 where France had a robot density of 132 robots per 10,000 workers.[46] Further, Figure 9 shows an increase in the hourly wage rate in the French manufacturing sector over time supporting the positive relationship found by Alonso et al. (2022) between robot density and real wages.

### 4.2.2 The Elasticity of Substitution Between the Two Types of High-Skilled Workers

We are mainly interested in the elasticity of substitution between the two types of high-skilled workers, $\sigma_3$. Unfortunately, there is no empirical literature that investigates the relationship between automation and the elasticity of substitution within the high-skilled workers we have introduced in Section 3. We ourselves are unable to provide an empirical estimate due to the unavailability of the data or time series required to make such a claim for these types of high-skilled workers.[47] The contribution of this section is therefore to highlight the importance of $\sigma_3$ as its value can result in two different outcomes depending on whether its value lies above or below one.

[44] As industrial robots are a subset of automation, they can be a useful indicator for the "AI revolution."

[45] Robot density is measured as the number of robots per workers.

[46] `https://ifr.org/ifr-press-releases/news/robot-density-rises-globally`

[47] To make an estimate, one would need labour input data for both types of high-skilled workers which is not currently available due to AI-based education only just starting to be implemented into the education sector.

Based on the data available which can be recovered from international reports, one can make an educated guess about the elasticity of substitution between these two types of high-skilled workers to better understand the impact automation will have. For example, the European Commission Report on the changing nature of work and skills finds an increasing shift in demand for digital and non-cognitive skills within the EU, specifically within the EU-28 group (Pabollet et al., 2019). The report projects information technology (IT), science, and engineering jobs to grow by 16%, other high-skilled jobs such as doctors, to grow by 15% while skilled manual labour is projected to decline by 2% between 2016-2030 in the EU-28. From this Pabollet et al. (2019) conclude automation will disproportionately favour those with higher education and basic digital skills:

> "*The greater capacity for data collection, processing and analytics, paired with machine learning and AI, entails tasks that require more analytical and digital skills from workers.*" (Pabollet et al., 2019, p. 29).

Pabollet et al. (2019) also find the probability of being in a high-paying job is higher for those who possess both moderate or advanced digital skills and non-cognitive skills. Their findings are replicated in Figure 11. They further find a larger wage premium when these skills are combined compared to workers who are not equipped with these skills. Figure 11 showcases this as we can see around 32% of workers who possess both moderate/advanced digital skills and non-routine skills are in the top quartile of the wage distribution. In contrast, almost 16% of workers with little or no digital skills but have non-cognitive skills are in the same wage quartile. This might be interpreted as the high-skilled workers with AI-based education are substitutable for high-skilled workers with traditional education, that is $\sigma_3 > 1$.

# 5 Discussions for Future Research

One avenue this research did not explore regarding skill-biased technological change is the impact automation will have on younger/older workers. The world, in general, is experiencing an ageing workforce and so whether automation complements younger and/or older workers has important policy implications (Acemoglu and Restrepo, 2022). Battisti and Gravina (2021) contribute to this area with their findings. For the period 1994-2005, using data for a number of OECD countries and two-digit level industries, Battisti and Gravina (2021) find a higher substitutability between robots and younger workers (aged 15-49) compared to the substitutability between robots and older workers (aged 50+).[48] Acemoglu

[48] The authors use a four-factor CES production model which consistently finds this relationship. They also find workers aged between 15 and 29 to be the most replaceable by robots. This finding could be explained

and Restrepo (2022) take a similar approach by analysing the relationship between demographics and automation. They argue ageing populations result in greater automation due to the shortage of middle-aged workers who specialise in manual production tasks.[49] Here middle-aged manufacturing workers are highly substitutable with industrial automation as this automation is designed to replenish the existing shortage. Thus, sectors that rely heavily on middle-aged workers will be impacted by automation more. It would be interesting to extend our model to incorporate this separation framework based on age. This would allow for an analysis of the impact of automation based on age and education levels further helping us to understand who the most vulnerable group of workers will be in the face of automation adoption.

A limitation of this paper is the current inability to quantitatively evaluate our model due to the lack of relevant data. This means the most pressing question regarding automation – the impact automation will have on labour markets – is unable to be empirically answered at this point in time. This leaves room for future research. Already data is being collected on investments in industrial robots (see among others Benmelech and Zator, 2022; Figures 2, 6-8, and 10 also show the IFR is collecting this data). However, in order to evaluate the full picture, data will need to be collected on the other types of automation, for example, AI. Data will also need to be collected regarding AI tools in education in order to evaluate our model.

Additionally, analysis of the impact of automation within developing countries could provide a robustness check of our assumption for our model - that automation and low-skilled labour are perfect substitutes. For example, Tang et al. (2021) examine the impact of robot adoption by firms in China using firm-level data. The authors find there is no statistically significant evidence that the share of low-skilled workers in robot adoption firms decreases (which we would expect based on the perfect substitute assumption) or increases relative to non-adoption firms.[50] In other words, "robots do not crowd out low-skilled workers because they are still irreplaceable in many labour-intensive Chinese firms." (Tang et al., 2021 p. 2).[51] Recently, Calì and Presidente (2002) use data on Indonesian manufacturing firms to document a positive impact on employment of adopting robots. This finding contrasts

by either i) the role of experience, ii) older workers may be less substitutable by robots as they tend not to deal with challenging manual tasks, or iii) human capital accumulation decay of the new generations (Battisti and Gravina, 2021, p. 3).

[49] Their theoretical model and empirical findings support this. They also find further supporting evidence of this as they do not find similar effects of aging on other technologies.

[50] They do find the share of high-skilled workers in robot adoption firms increase relative to non-adoption firms which is consistent with what we would expect based on the literature.

[51] The authors also distinguish workers by education and note education levels are strongly correlated with skill levels. They find the shares of highly educated workers increase significantly with robot adoption and shares of less educated workers decrease.

with the existing evidence of negative impacts in economies at relatively advanced stages of automation, and could be explained by diminishing returns to robots, given the relatively low robot adoption in Indonesia.[52] These two studies would suggest that the assumption used in our model – automation and low-skilled workers are perfect substitutes – may not hold in a developing country setting. In contrast, Berg et al.'s (2018) findings suggest this assumption may hold in advanced countries. They find that as robots substitute more easily with workers, GDP per capita increases more but at the expense of the labour share declining with low-skilled workers facing the greatest decrease in the labour share and relative wages. Hence, testing this assumption is important because the impact of automation on developing countries may differ from the impact experienced by advanced countries. This will have important policy implications for economic development, wage inequality, growth, and trade.

Another area in the literature that requires future research is AI and trade. Considering the world economy has increasingly become more open, the impact AI and automation may have on international trade is an important area to understand. So far, an important contribution in this area comes from Goldfarb and Trefler (2019). To guide future research, the authors suggest extending the superstar model with heterogenous scientists to incorporate endogenous growth models in a trade setting. This would allow for the inclusion of knowledge creation within the firm or knowledge diffusion across borders. Their superstar model would also need to be extended to incorporate the geography of the industry and be able to consider whether the scale of returns is external to the firm. Aspects of our model – such as the composite high-skilled labour – can be incorporated into a trade model which would extend the literature by analysing the implications of AI and education in conjunction with AI and trade. These characteristics are important to model as the results will have important trade policy implications.

# 6 Conclusions

The rise in automation has sparked a rampant debate regarding the future of work. Some argue the fear elicited from the idea that automation will take our jobs is unwarranted (Salmon, 2019; Diamond, 2020; Benmelech and Zator, 2022) while others argue the impact of automation could be significant due to the potential widespread loss of jobs (Frey and Osborne, 2017; World Bank, 2016, 2019). Despite the opposing views, it is clear that automation will lead to a displacement of some tasks but will also see the creation of new tasks.

This paper seeks to extend the existing literature on automation and the future of work

[52] `https://voxeu.org/article/robots-economic-development`

by incorporating AI-based education into a theoretical model. By using a variation of the model used in Heer and Irmen (2019), Lankisch et al. (2019), and Prettner (2019) and extending it to include composite high-skilled labour, our model is able to demonstrate several propositions. Firstly, our model demonstrates that the accumulation of automation will lead to a rise in the skill premium and the AI skill premium, thus contributing to a rise in wage inequality. Secondly, our model demonstrates that perpetual long-run economic growth is possible despite there being no exogenous technological change. These results align with the findings of Heer and Irmen (2019), Lankisch et al. (2019), and Prettner (2019) who argue increasing automation will result in an increase in wage inequality.

This paper also analyses the role automation plays in SBTC. Although we are unable to provide empirical evidence to assess our model regarding SBTC and capital-skill complementarity, our model demonstrates the importance of the elasticity of substitution between the two types of high-skilled workers, $\sigma_3$. Dependent on the value of $\sigma_3$, technical change will either favour high-skilled workers with a traditional education or it will favour high-skilled workers with an AI-based education background. An educated guess, based on past technological advancements, would assume that automation would favour high-skilled workers with an AI-based education background as they possess the necessary technological skills to complement automation. However, an empirical analysis of this is left for future research.

The inclusion of AI-based education in our model highlights an important policy implication. In line with our findings, investment in AI tools for the education sector (investment in higher education) may be a way to combat any disruptive impact of automation. By investing in AI-based education, the share of high-skilled workers with an AI-based education background will increase. As our model highlights, these high-skilled workers were the least impacted by automation. This type of policy would aid in the reduction of wage inequality as a result of automation. This policy recommendation is reinforced by previous literature (Goos, 2018; Lankisch et al., 2019).

In order to ensure workers can remain in the labour force despite the task in their job becoming fully automated, would require an adequate retraining policy that ensures unskilled workers or high-skilled workers with a traditional education background can become high-skilled workers with an AI-based education.[53] Retraining programmes could, for example, focus on teaching and training workers on non-cognitive skills such as social skills. These skills are currently the hardest to automate and thus jobs requiring these skills may increase as automation diffuses into society.

Other policies such as income redistribution policies and innovation policies which aim to ensure automation complements workers' skills have been suggested in the literature (see

[53]See Jaimovich et al. (2021) for an in-depth analysis of different policies regarding automation.

Goos, 2018). However, further analysis into the quantitative effects of automation on society is required to ensure the best policies are implemented.

# References

Acemoglu, D. 1998. Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality. Quarterly Journal of Economics, 113(4), 1055-1089.

Acemoglu, D. 2002a. Directed Technical Change. Review of Economic Studies, 69, 781-809.

Acemoglu, D. 2002b. Technical Change, Inequality, and the Labor Market. Journal of Economic Literature, 40(1), 7-72.

Acemoglu D. 2009. Introduction to Modern Economic Growth. Princeton, New Jersey: Princeton University Press.

Acemoglu, D., Autor, D. 2011. Skills, Tasks and Technologies: Implications for Employment and Earnings. In Ashenfelter, O., Card, D. (eds.), Handbook of Labor Economics Volume 4B, Elsevier, 1043-1171.

Acemoglu, D., Autor, D. 2012. What Does Human Capital Do? A Review of Goldin and Katz's *The Race between Education and Technology*. Journal of Economic Literature, 50(2), 426-463.

Acemoglu, D., Restrepo, P. 2019. Automation and New Tasks: How Technology Displaces and Reinstates Labor. Journal of Economic Perspectives, 33(2), 3-30.

Acemoglu, D., Restrepo, P. 2020. Robots and Jobs: Evidence from US Labor Markets. Journal of Political Economy, 128(6), 2188-2244.

Acemoglu, D., Restrepo, P. 2022. Demographics and Automation. Review of Economic Studies, 89(1), 1-44.

Agrawal, A., Gans, J., Goldfarb, A. 2019. Economic Policy for Artificial Intelligence. Innovation Policy and the Economy, 19, 139-159.

AI Forum of New Zealand. 2018. Artificial Intelligence: Shaping a Future New Zealand. AI Forum of New Zealand Report.
https://aiforum.org.nz/wp-content/uploads/2018/07/AI-Report-2018_web-version.pdf

Alonso, C., Berg, A., Kothari, S., Papageorgiou, C., Rehman, S. 2022. Will the AI Revolution Cause a Great Divergence? Journal of Monetary Economics, In press. https://doi.org/10.1016/j.jmoneco.2022.01.004

Angrist, J. D. 1995. The Economic Returns to Schooling in the West Bank and Gaza Strip. American Economic Review, 85(5), 1065-1087.

Autor, D. H. 2014. Skills, Education, and the Rise of Earnings Inequality Among the "Other 99 Percent". Science, 344(6186), 843-851. https://economics.mit.edu/files/11554

Autor, D. H., Katz, L. F., Kearney, M. S. 2008. Trends in U.S. Wage Inequality: Revising the Revisionists. Review of Economics and Statistics, 90(2), 300-323.

Bartel, A. P., Lichtenberg, F. R. 1987. The Comparative Advantage of Educated Workers in Implementing New Technology. Review of Economics and Statistics, 69(1), 1-11.

Battisti, M., Gravina, A. F. 2021. Do Robots Complement or Substitute for Older Workers? Economics Letters, 208, Article 110064.

Benmelech, E., Zator, M. 2022. Robots and Firm Investment. NBER Working Paper 29676, https://www.nber.org/papers/w29676

Berg, A., Buffie, E. F., Zanna, L.-F. 2018. Should We Fear the Robot Revolution? (The Correct Answer is Yes). Journal of Monetary Economics, 97, 117-148.

Berman, E., Bound, J., Griliches, Z. 1994. Changes in the Demand for Skilled Labor within U.S. Manufacturing: Evidence from the Annual Survey of Manufactures. Quarterly Journal of Economics, 109(2), 367-397.

Borjas, G. J. 2003. The Labour Demand Curve is Downward Sloping: Reexamining the Impact of Immigration on the Labour Market. Quarterly Journal of Economics, 118(4), 1335-1374.

Calì, M., Presidente, G. 2022. Robots for Economic Development. Working Paper. http://hdl.handle.net/10419/249581

Cassell, J., Bai, Z., Paranjape, B., Sinha, T., Slebodnick, D., Yi Tan, L. 2016. Sensing Curiosity in Play and Responding. The ArticuLab, Carnegie Mellon University. http://articulab.hcii.cs.cmu.edu/projects/scipr/

Ciccone, A., Peri, G. 2005. Long-Run Substitutability Between More and Less Educated Workers: Evidence from U.S. States, 1950-1990. Review of Economics and Statistics, 87(4), 652-663.

Cummins, J. G., Violante, G. L. 2002. Investment-Specific Technical Change in the United States (1947-2000): Measurement and Macroeconomic Consequences. Review of Economic Dynamics, 5(2), 243-284.

Diamond, A. M. 2020. Robots and Computers Enhance Us More Than They Replace Us. The American Economist, 65(1), 4-10.

Duggan, S. 2020. AI in Education: Change at the Speed of Learning. UNESCO Institute for Information Technologies in Education (IITE) Policy Brief. https://iite.unesco.org/wp-content/uploads/2020/11/Steven_Duggan_AI-in-Education_2020.pdf

Foster, A. D., Rosenzweig, M. R. 1995. Learning by Doing and Learning from Others: Human Capital and Technical Change in Agriculture. Journal of Political Economy, 103(6), 1176-1209.

Frey, C. B., Osborne, M. A. 2017. The Future of Employment: How Susceptible are Jobs to Computerisation? Technological Forecasting and Social Change, 114, 254-280.

Galor, O., Moav, O. 2000. Ability-Biased Technological Transition, Wage Inequality, and Economic Growth. Quarterly Journal of Economics, 115(2), 469-497.

Gasteiger, E., Prettner, K. 2022. Automation, Stagnation, and the Implications of a Robot Tax. Macroeconomic Dynamics, 26(1), 218-249.

Gavaghan, C., Knott, A., Maclaurin, J. 2021. The Impact of Artificial Intelligence on Jobs and Work in New Zealand. (Commissioned Report for External Body). University of Otago. http://hdl.handle.net/10523/12124

Goldfarb, A., Trefler, D. 2019. Artificial Intelligence and International Trade. In A. Agrawal, J. Gans, A. Goldfarb (eds.), The Economics of Artificial Intelligence: An Agenda. Chicago: The University of Chicago Press, 463-492.

Goos, M. 2018. The Impact of Technological Progress on Labour Markets: Policy Challenges. Oxford Review of Economic Policy, 34(3), 362-375.

Grant, R. 2022. The Better Educated, the Better Off? AI-Based Education and the Labour Force: Some Theoretical Considerations. Unpublished Master Thesis. University of Otago.

Hamermesh, D. S. 1995. Labor Demand and the Source of Adjustment Costs. Economic Journal, 105(430), 620-634.

Havranek, T., Irsova, Z., Laslopova, L., Zeynalova, O. 2021. Publication and Attenuation Bias in Measuring Skill Substitution. Unpublished Manuscript (21 December 2021 version). http://meta-analysis.cz/skill/skill.pdf

Heer, B., Irmen, A. 2019. Automation, Economic Growth, and the Labor Share – A Comment on Prettner (2019). CESifo Working Paper No.7730. https://ssrn.com/abstract=3422625

Hémous, D., Olsen, M. 2022. The Rise of the Machines: Automation, Horizontal Innovation, and Income Inequality. American Economic Journal: Macroeconomics, 14(1), 179-223.

Hornstein, A., Krusell, P. 2005. The Effect of Technical Change on Labour Market Inequalities. In Aghion, P., Durlauf, S. N. (eds.) Handbook of Economic Growth. Volume 1, Part B, 1275-1370.

Hornstein, A., Krusell, P., Violante, G. L. 2005. Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanism. Federal Reserve Bank of Richmond Economic Quarterly, 91(3), 19-51.

Hudson, J. 2019. The Robot Revolution: Understanding the Social and Economic Impact. Cheltenham: Edward Elgar Publishing.

Inada, K.-I. 1963. On a Two-Sector Model of Economic Growth: Comments and a Generalization. Review of Economic Studies, 30(2), 119-127.

Jaimovich, N., Saporta-Eksten, I., Siu, H., Yedid-Levi, Y. 2021. The Macroeconomics of Automation: Data, Theory, and Policy Analysis. Journal of Monetary Economics, 122, 1-16.

Jaumotte, F., Lall, S., Papageorgiou, C. 2008. Rising Income Inequality: Technology, or Trade and Financial Globalization? IMF Working Paper WP/08/185 https://www.imf.org/en/Publications/WP/Issues/2016/12/31/Rising-Income-Inequality-Technology-or-Trade-and-Financial-Globalization-22170

Jerzmanowski, M., Tamura, R. 2020. Aggregate Elasticity of Substitution Between Skills: Estimates from a Macroeconomic Approach. Munich Personal RePEc Archive Paper No. 100768. https://mpra.ub.uni-muenchen.de/100768/

Juhn, C., Murphy, K. M., Pierce, B. 1993. Wage Inequality and the Rise in Returns to Skill. Journal of Political Economy, 101(3), 410-442.

Katz, L. F., Murphy, K. M. 1992. Changes in Relative Wages, 1963-1987: Supply and Demand Factors. Quarterly Journal of Economics, 107(1), 35-78.

Lankisch, C. Prettner, K., Prskawetz, A. 2017. Robots and the Skill Premium: An Automation-Based Explanation of Wage Inequality. Hohenheim Discussion Papers in Business, Economics and Social Sciences, No. 29-2017.

Lankisch, C., Prettner, K., Prskawetz, A. 2019. How Can Robots Affect Wage Inequality? Economic Modelling, 81, 161-169.

Machin, S., Van Reenen, J. 1998. Technology and Changes in Skill Structure: Evidence from Seven OECD Countries. Quarterly Journal of Economics, 113(4), 1215-1244.

Milanesi, C. 2020. Can AI Drive Education Forward? Forbes. Retrieved from https://www.forbes.com/sites/carolinamilanesi/2020/01/23/can-ai-drive-education-forward/?sh=2b704c147964

Murphy, K. M., Welch, F. 1992. The Structure of Wages. Quarterly Journal of Economics, 107(1), 285-326.

Nelson, R. R., Phelps, E. S. 1966. Investment in Humans, Technological Diffusion, and Economic Growth. American Economic Review, 56(1/2), 69-75.

New Zealand Digital Skills Forum. 2018. Digital Skills For a Digital Nation. New Zealand Digital Skills Forum Report. https://dsfnz.wpengine.com/wp-content/uploads/2018/01/digital-skills-for-a-digital-nation-online.pdf

Pabollet, A., Cabrera Giraldez, E., Caena, F., et al. 2019. The Changing Nature of Work and Skills in the Digital Age. European Commission, Joint Research Centre Report. https://data.europa.eu/doi/10.2760/373892

Prettner, K. 2019. A Note on the Implications of Automation for Economic Growth and the Labor Share. Macroeconomic Dynamics, 23(3), 1294-1301.

Salmon, K. 2019. Jobs, Robots & Us: Why the Future of Work in New Zealand is in Our Hands. Wellington, New Zealand: Bridget Williams Books.

Tang, C., Huang, K., Liu, Q. 2021. Robots and Skill-Biased Development in Employment Structure: Evidence from China. Economics Letters, 205, Article 109960.

Violante, G. L. 2008. Skill-Biased Technical Change. In Palgrave Macmillan (eds.) The New Palgrave Dictionary of Economics. Palgrave Macmillan, London. https://doi.org/10.1057/978-1-349-95121-5_2388-1

Williams, R. 2019. Fan Hui: What I Learned from Losing to DeepMind's AlphaGo. https://inews.co.uk/news/technology/fan-hui-what-i-learned-from-losing-to-deepminds-alphago-google-295005

World Bank. 2016. World Development Report: Digital Dividends. World Bank Group, Washington DC. https://www.worldbank.org/en/publication/wdr2016

World Bank. 2019. World Development Report 2019: The Changing Nature of Work. World Bank Group, Washington DC. https://documents1.worldbank.org/curated/en/816281518818814423/pdf/2019-WDR-Report.pdf

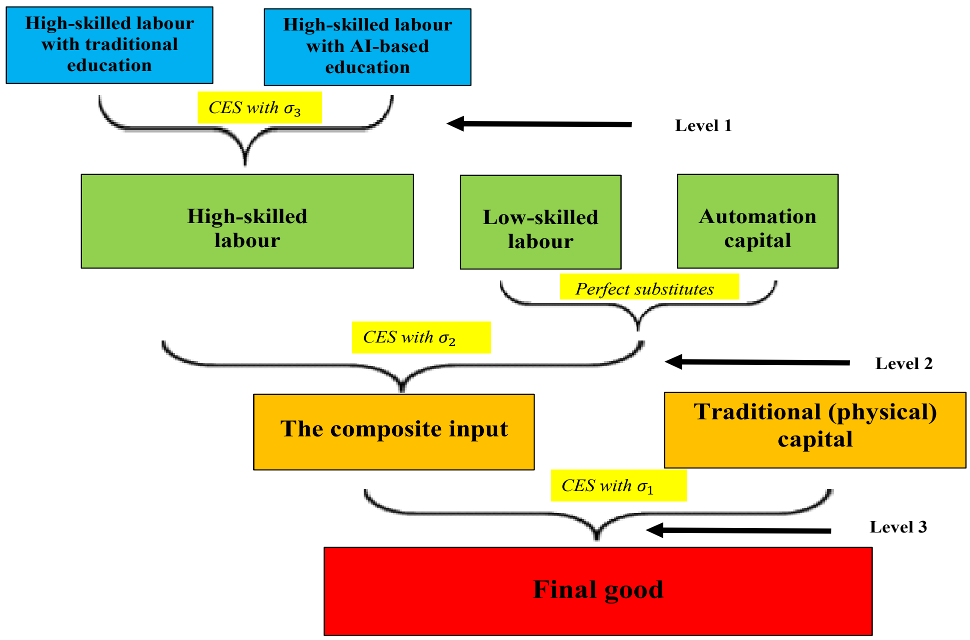


Figure 1: A Three-Level Constant Elasticity of Substitution (CES) Production Technology

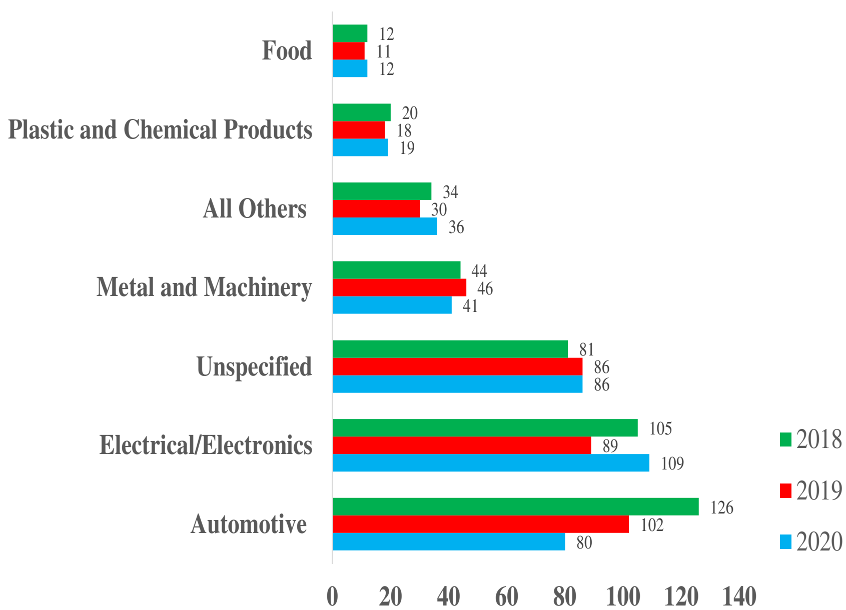


Figure 2: Annual Installations of Industrial Robots by Customer Industry, World*

*In 1000 units.

Source: `https://ifr.org/img/worldrobotics/Executive_Summary_WR_Industrial_Robots_2021.pdf`

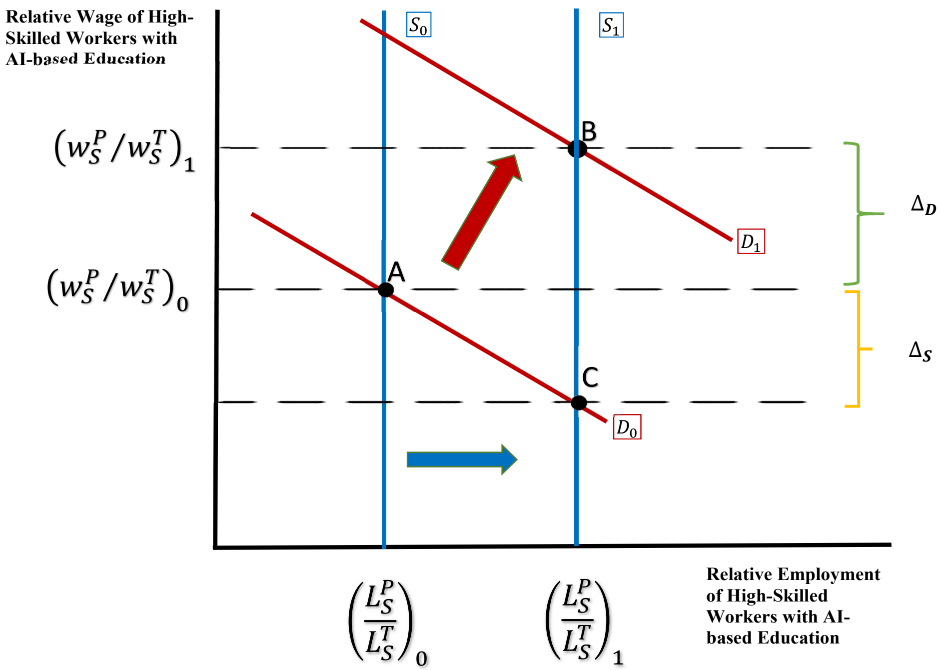


Figure 3: SBTC in the Context of Differentiation within High-Skilled Workers

Source: Authors' own illustration.

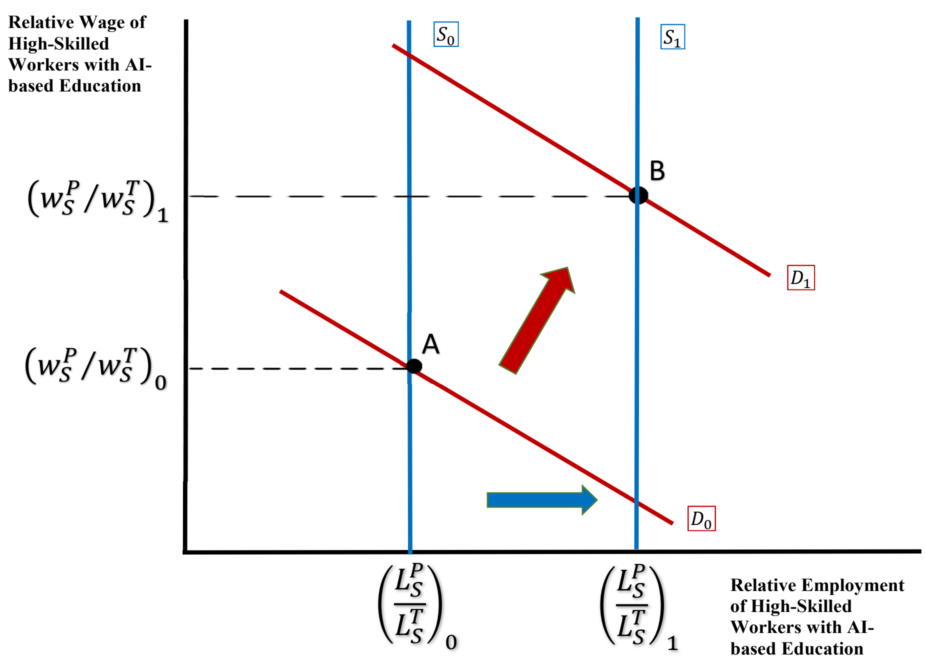


Figure 4: SBTC in the Context of Differentiation within High-skilled Workers, $\sigma_3 > 1$

Source: Authors' own illustration.

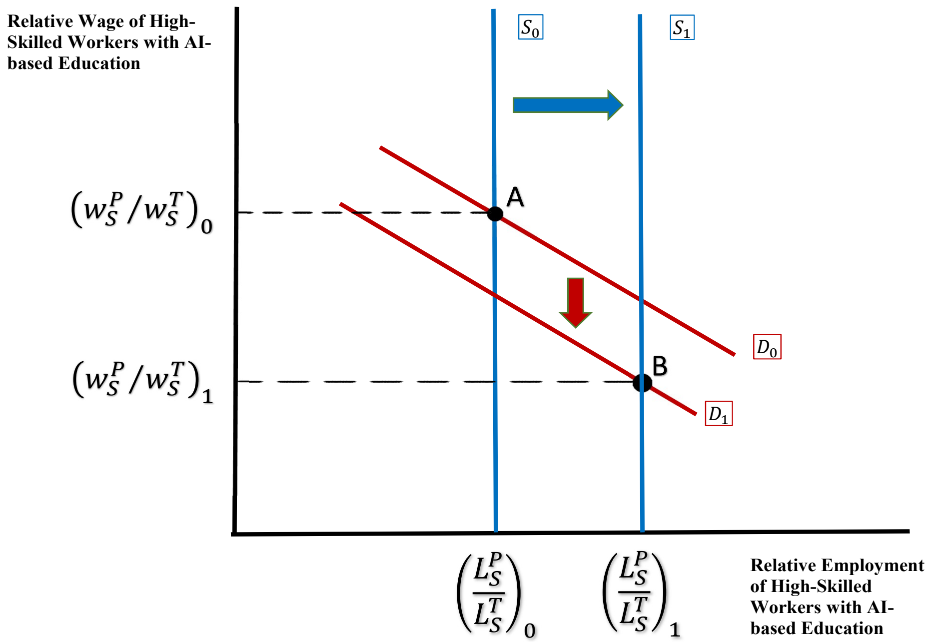


Figure 5: SBTC in the Context of Differentiation within High-Skilled Workers, $\sigma_3 < 1$

Source: Authors' own illustration.

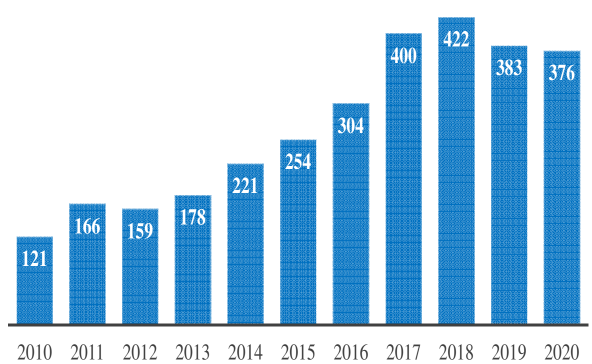


Figure 6: Annual Installations of Industrial Robots, World*

*In 1000 units.

Source: `https://ifr.org/downloads/press2018/New_installations_WR2021.jpg`

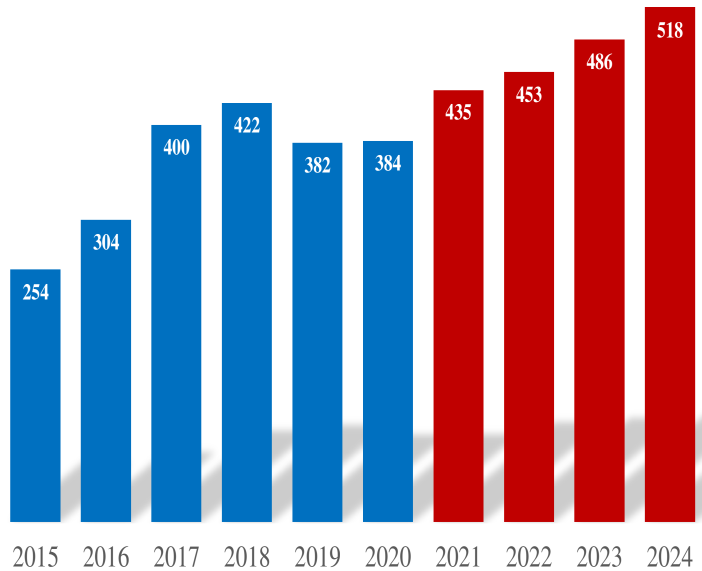


Figure 7: Annual Installations of Industrial Robots, 2015-2020 vs. 2021-2024*

*In 1000 units. The figures for the 2021-2024 period are based on forecasts.

Source: `https://ifr.org/downloads/press2018/Forecast_installations_WR2021.jpg`

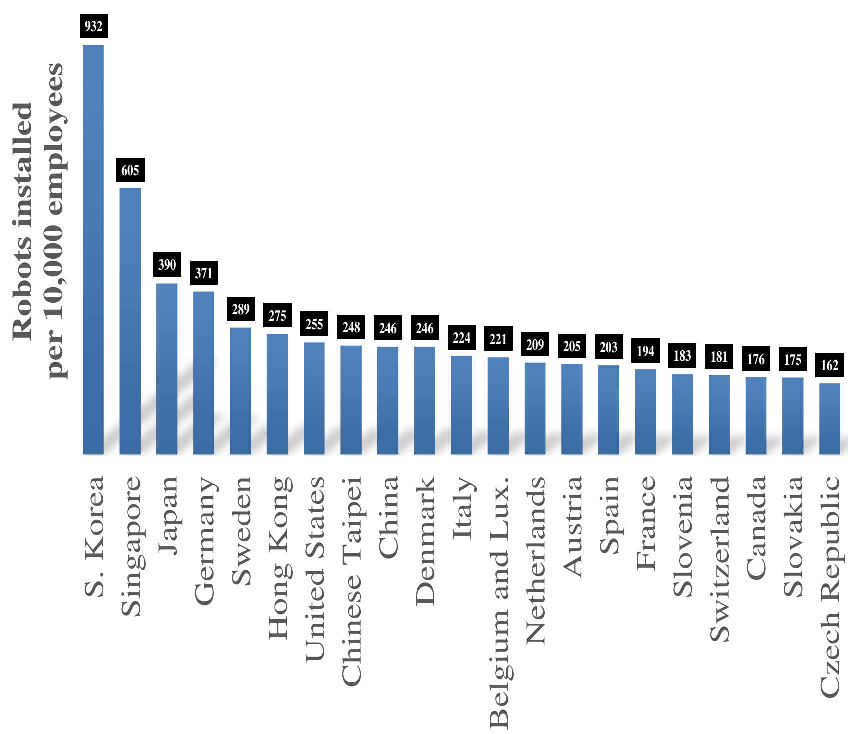


Figure 8: Robot Density in the Manufacturing Industry, 2020*

Source: `https://ifr.org/ifr-press-releases/news/robot-density-nearly-doubled-globally`

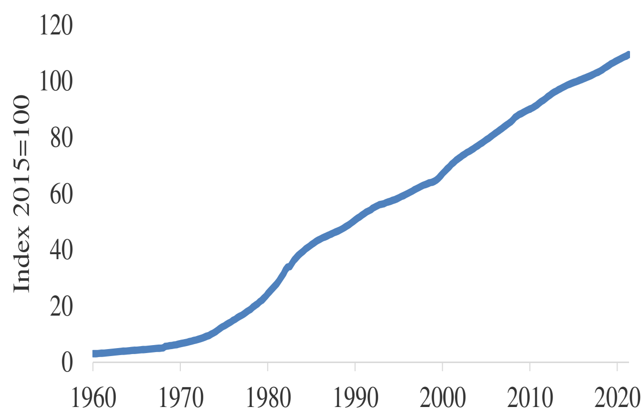


Figure 9: Hourly Wage Rate in Manufacturing, France, 1960-2021*

*Quarterly and seasonally adjusted data.

Source: OECD, Hourly Earnings: Manufacturing for France [LCEAMN01FRQ661S] Retrieved from FRED, Federal Reserve Bank of St. Louis, December 22, 2021. `https://fred.stlouisfed.org/series/LCEAMN01FRQ661S`

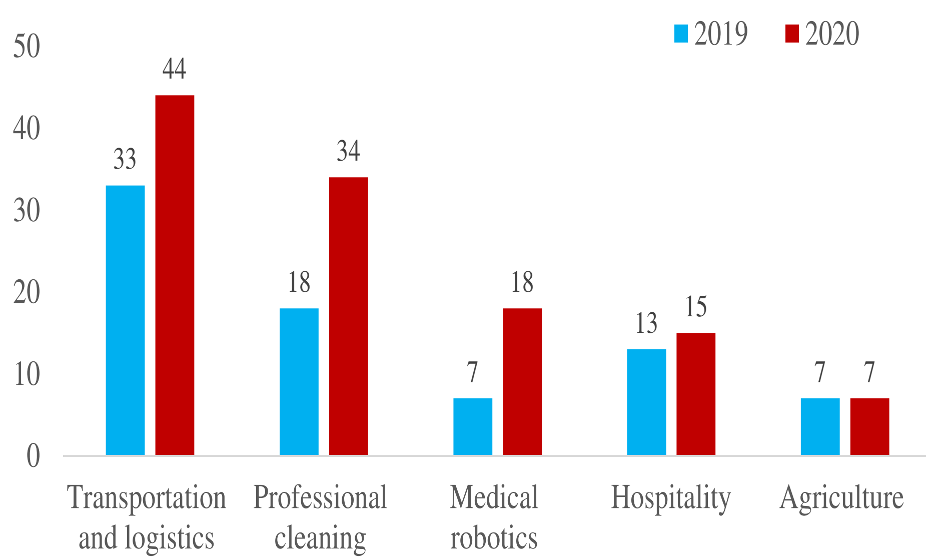


Figure 10: Service Robots for Professional Use: Top 5 Applications, Unit Sales

*In 1000 units.

Source: `https://ifr.org/ifr-press-releases/news/service-robots-hit-double-digit-growth-worldwide`

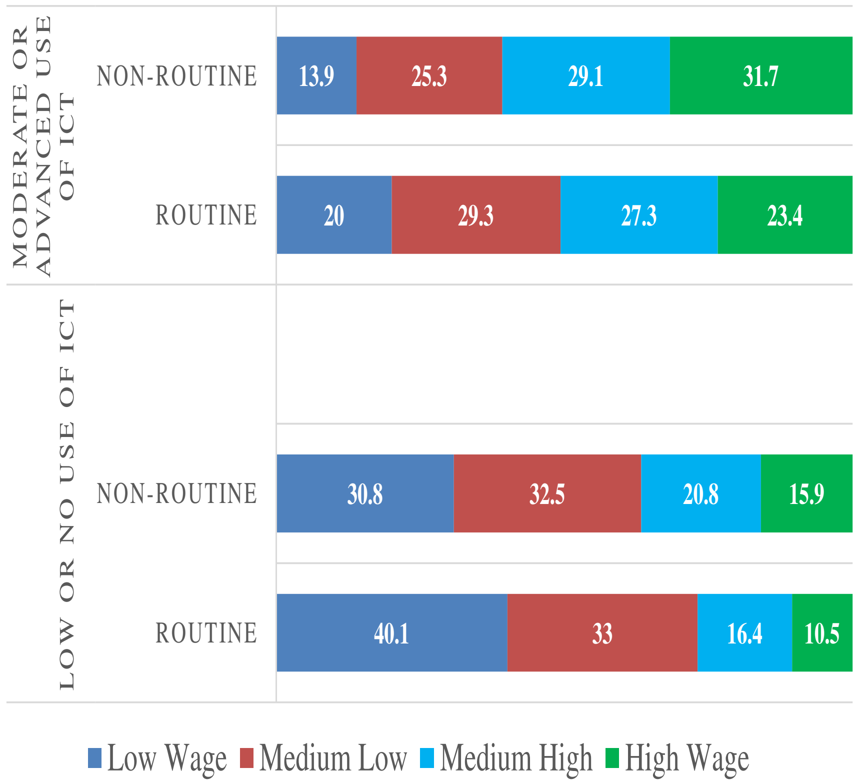


Figure 11: Wage Distribution Probability Based on Digital and Non-Cognitive Skills

Source: Replicated from Pabollet et al. (2019, Figure 14).

# Appendix A

## Appendix A.1 Technical Results and Omitted Proofs

This appendix contains the proofs and derivations omitted from the main body of this paper.
**Proof of Proposition 1**: Combining Equations 5(a)-5(c) with Lemma 2 yields

$$\left(a\frac{Y(t)}{K(t)}\right)^{\frac{1}{\sigma_1}} = \left((1-a)\frac{Y(t)}{V(t)}\right)^{\frac{1}{\sigma_1}} \left((1-b)\frac{V(t)}{L_u(t)+P(t)}\right)^{\frac{1}{\sigma_2}}. \tag{A1}$$

Rearranging (A1) gives the following useful ratios:

$$\frac{L_u(t)+P(t)}{V(t} = \left(\frac{1-a}{a}\right)^{\frac{\sigma_2}{\sigma_1}}(1-b)\left(\frac{K(t)}{V(t)}\right)^{\frac{\sigma_2}{\sigma_1}}. \tag{A2}$$

$$\frac{K(t)}{V(t)} = \left(\frac{a}{1-a}\right)\left(\frac{1}{1-b}\right)^{\frac{\sigma_1}{\sigma_2}}\left(\frac{L_u(t)+P(t)}{V(t}\right)^{\frac{\sigma_1}{\sigma_2}}. \tag{A3}$$

These ratios, (A2) and (A3), can be used in Equation (1) to obtain

$$\begin{aligned}
\frac{Y(t)}{V(t)} &= \left[a^{\frac{1}{\sigma_1}}\left(\frac{K(t)}{V(t)}\right)^{\frac{\sigma_1-1}{\sigma_1}} + (1-a)^{\frac{1}{\sigma_1}}\right]^{\frac{\sigma_1}{\sigma_1-1}} \\
&= \left[a^{\frac{1}{\sigma_1}}\left(\frac{a}{1-a}\right)^{\frac{\sigma_1-1}{\sigma_1}}\left(\frac{1}{1-b}\right)^{\frac{\sigma_1-1}{\sigma_2}}\left(\frac{L_u(t)+P(t)}{V(t)}\right)^{\frac{\sigma_1-1}{\sigma_2}} + (1-a)^{\frac{1}{\sigma_1}}\right]^{\frac{\sigma_1}{\sigma_1-1}} \\
&= \left[\left(\frac{a}{1-a}\right)(1-a)^{\frac{1}{\sigma_1}}\left(\frac{1}{1-b}\right)^{\frac{\sigma_1-1}{\sigma_2}}\left(\frac{L_u(t)+P(t)}{V(t)}\right)^{\frac{\sigma_1-1}{\sigma_2}} + (1-a)^{\frac{1}{\sigma_1}}\right]^{\frac{\sigma_1}{\sigma_1-1}} \\
&= (1-a)^{\frac{1}{\sigma_1-1}}\left[\left(\frac{a}{1-a}\right)\left(\frac{1}{1-b}\right)^{\frac{\sigma_1-1}{\sigma_2}}\left(\frac{L_u(t)+P(t)}{V(t)}\right)^{\frac{\sigma_1-1}{\sigma_2}} + 1\right]^{\frac{\sigma_1}{\sigma_1-1}} \\
&= (1-a)^{\frac{1}{\sigma_1-1}}\left(\frac{1}{1-a}\right)^{\frac{\sigma_1}{\sigma_1-1}}\left[a\left(\frac{1}{1-b}\right)^{\frac{\sigma_1-1}{\sigma_2}}\left(\frac{L_u(t)+P(t)}{V(t)}\right)^{\frac{\sigma_1-1}{\sigma_2}} + (1-a)\right]^{\frac{\sigma_1}{\sigma_1-1}} \\
&= \left(\frac{1}{1-a}\right)\left[a\left(\frac{1}{1-b}\right)^{\frac{\sigma_1-1}{\sigma_2}}\left(\frac{L_u(t)+P(t)}{V(t)}\right)^{\frac{\sigma_1-1}{\sigma_2}} + (1-a)\right]^{\frac{\sigma_1}{\sigma_1-1}}.
\end{aligned} \tag{A4}$$

Using (A2) and (A4) in Equation 5(c) we obtain

$$\begin{aligned}
w_u(t) &= \left((1-a)\frac{Y(t)}{V(t)}\right)^{\frac{1}{\sigma_1}}\left((1-b)\frac{V(t)}{L_u(t)+P(t)}\right)^{\frac{1}{\sigma_2}}\\
&= \left[a\left(\frac{1}{1-b}\right)^{\frac{\sigma_1-1}{\sigma_2}}\left(\frac{L_u(t)+P(t)}{V(t)}\right)^{\frac{\sigma_1-1}{\sigma_2}}+(1-a)\right]^{\frac{1}{\sigma_1-1}}\left((1-b)\frac{V(t)}{L_u(t)+P(t)}\right)^{\frac{1}{\sigma_2}}\\
&= \left[a+(1-a)\big(1-b\big)^{\frac{\sigma_1-1}{\sigma_2}}\left(\frac{L_u(t)+P(t)}{V(t)}\right)^{\frac{1-\sigma_1}{\sigma_2}}\right]^{\frac{1}{\sigma_1-1}}\\
&= \left[a+(1-a)\big(1-b\big)^{\frac{\sigma_1-1}{\sigma_2}}\big(L_u(t)+P(t)\big)^{\frac{1-\sigma_1}{\sigma_2}}\big(V(t)\big)^{\frac{\sigma_1-1}{\sigma_2}}\right]^{\frac{1}{\sigma_1-1}}. \qquad \text{(A5)}
\end{aligned}$$

Using the expression for the composite input, $V(t)$, which is given by Equation (2) in (A5) and rearranging we obtain

$$w_u(t) = \left[a+(1-a)\big(1-b\big)^{\frac{\sigma_1-1}{\sigma_2}}\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}\big(L_u(t)+P(t)\big)^{\frac{1-\sigma_2}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\right]^{\frac{\sigma_1-1}{\sigma_2-1}}\right]^{\frac{1}{\sigma_1-1}}. \qquad \text{(A6)}$$

The effect of an increase in the stock of industrial robots on the wages of low-skilled workers is given by

$$\frac{\partial w_u(t)}{\partial P(t)} = -\frac{1}{\sigma_2}(1-a)(1-b)^{\frac{\sigma_1-1}{\sigma_2}}b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_1-1}{\sigma_2}}\big[L_u(t)+P(t)\big]^{\frac{1-2\sigma_2}{\sigma_2}}Z(t), \qquad \text{(A7)}$$

where

$$\begin{aligned}
Z(t) \equiv &\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}[L_u(t)+P(t)]^{\frac{1-\sigma_2}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\right]^{\frac{\sigma_1-\sigma_2}{\sigma_2-1}}\\
&\times\left[a+(1-a)\big(1-b\big)^{\frac{\sigma_1-1}{\sigma_2}}\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}[L_u(t)+P(t)]^{\frac{1-\sigma_2}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\right]^{\frac{\sigma_1-1}{\sigma_2-1}}\right]^{\frac{2-\sigma_1}{\sigma_1-1}}.
\end{aligned}$$

The sign of $\frac{\partial w_u(t)}{\partial P(t)}$ is determined by the minus sign in front of the expression on the right-hand side of (A7), because all other components of (A7) have positive signs. Thus, $\frac{\partial w_u(t)}{\partial P(t)} < 0$.

**Proof of Proposition 2**: Using (A4) in the expression for wages of high-skilled workers with a traditional education background, Equation (5d), yields

$$\begin{aligned}
w_s^T(t) &= \left((1-a)\frac{Y(t)}{V(t)}\right)^{\frac{1}{\sigma_1}}\left(b\frac{V(t)}{L_s(t)}\right)^{\frac{1}{\sigma_2}}\left(c\frac{L_s(t)}{L_s^T(t)}\right)^{\frac{1}{\sigma_3}}\\
&= \left[a\left(\frac{1}{1-b}\right)^{\frac{\sigma_1-1}{\sigma_2}}\left(\frac{L_u(t)+P(t)}{V(t)}\right)^{\frac{\sigma_1-1}{\sigma_2}}+(1-a)\right]^{\frac{1}{1-\sigma_1}}\left(b\frac{V(t)}{L_s(t)}\right)^{\frac{1}{\sigma_2}}\left(c\frac{L_s(t)}{L_s^T(t)}\right)^{\frac{1}{\sigma_3}}\\
&= \left[a\big(1-b\big)^{\frac{1-\sigma_1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_1-1}{\sigma_2}}+(1-a)\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}+\big(1-b\big)^{\frac{1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_2}{\sigma_2-1}}\right]^{\frac{1}{1-\sigma_1}}\\
&\times\left(\frac{b}{L_s(t)}\right)^{\frac{1}{\sigma_2}}\left(c\frac{L_s(t)}{L_s^T(t)}\right)^{\frac{1}{\sigma_3}}. \qquad \text{(A8)}
\end{aligned}$$

The last expression is obtained by rearranging the expression for the composite input, $V(t)$, which is given by Equation (2). The effect of an increase in the stock of industrial robots on the wages of high-skilled workers with a traditional education background is given by

$$\frac{\partial W_s^T(t)}{\partial P(t)} = \Bigg[a(1-b)^{\frac{1-\sigma_1}{\sigma_2}}\left(\frac{\sigma_1-1}{\sigma_2}\right)\left[L_u(t)+P(t)\right]^{\frac{\sigma_1-\sigma_2-1}{\sigma_2}}$$
$$+(1-a)(1-b)^{\frac{1}{\sigma_2}}\left(\frac{\sigma_1-1}{\sigma_2}\right)\left[L_u(t)+P(t)\right]^{\frac{-1}{\sigma_2}}\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_1-1}{\sigma_2-1}}\Bigg]$$
$$\times\frac{1}{\sigma_1-1}\left[a(1-b)^{\frac{1-\sigma_1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_1-1}{\sigma_2}}+(1-a)\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_1-1}{\sigma_2-1}}\right]^{\frac{2-\sigma_1}{\sigma_1-1}}$$
$$\times\left(\frac{b}{L_s(t)}\right)^{\frac{1}{\sigma_2}}\left(c\frac{L_s(t)}{L_s^T(t)}\right)^{\frac{1}{\sigma_3}}$$
$$=\frac{1}{\sigma_2}\Bigg[a(1-b)^{\frac{1-\sigma_1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_1-\sigma_2-1}{\sigma_2}}$$
$$+(1-a)(1-b)^{\frac{1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{-1}{\sigma_2}}\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_1-1}{\sigma_2-1}}\Bigg]$$
$$\times\left[a(1-b)^{\frac{1-\sigma_1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_1-1}{\sigma_2}}+(1-a)\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_1-1}{\sigma_2-1}}\right]^{\frac{2-\sigma_1}{\sigma_1-1}}$$
$$\times\left(\frac{b}{L_s(t)}\right)^{\frac{1}{\sigma_2}}\left(c\frac{L_s(t)}{L_s^T(t)}\right)^{\frac{1}{\sigma_3}}. \tag{A9}$$

The sign of $\frac{\partial W_s^T(t)}{\partial P(t)}$ is determined by the sign of $\frac{1}{\sigma_2}$, which is positive. Thus, $\frac{\partial W_s^T(t)}{\partial P(t)} > 0$. Similarly, using (A4) in the expression for wages of high-skilled workers with an AI-based education background, Equation (5e), yields

$$w_s^P(t) = \left((1-a)\frac{Y(t)}{V(t)}\right)^{\frac{1}{\sigma_1}}\left(b\frac{V(t)}{L_s(t)}\right)^{\frac{1}{\sigma_2}}\left((1-c)\frac{L_s(t)}{L_s^P(t)}\right)^{\frac{1}{\sigma_3}}$$
$$=\left[a\left(\frac{1}{1-b}\right)^{\frac{\sigma_1-1}{\sigma_2}}\left(\frac{L_u(t)+P(t)}{V(t)}\right)^{\frac{\sigma_1-1}{\sigma_2}}+(1-a)\right]^{\frac{1}{1-\sigma_1}}\left(b\frac{V(t)}{L_s(t)}\right)^{\frac{1}{\sigma_2}}\left((1-c)\frac{L_s(t)}{L_s^P(t)}\right)^{\frac{1}{\sigma_3}}$$
$$=\left[a(1-b)^{\frac{1-\sigma_1}{\sigma_2}}\left(L_u(t)+P(t)\right)^{\frac{\sigma_1-1}{\sigma_2}}+(1-a)\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\left(L_u(t)+P(t)\right)^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_2}{\sigma_2-1}}\right]^{\frac{1}{1-\sigma_1}}$$
$$\times\left(\frac{b}{L_s(t)}\right)^{\frac{1}{\sigma_2}}\left((1-c)\frac{L_s(t)}{L_s^P(t)}\right)^{\frac{1}{\sigma_3}}. \tag{A10}$$

The last expression is obtained by rearranging the expression for the composite input, $V(t)$, which is given by Equation (2). The effect of an increase in the stock of industrial robots on the wages of high-skilled workers with an AI-based education background is given by

$$
\begin{aligned}
\frac{\partial W_s^P(t)}{\partial P(t)} &= \Bigg[a(1-b)^{\frac{1-\sigma_1}{\sigma_2}}\left(\frac{\sigma_1-1}{\sigma_2}\right)\left[L_u(t)+P(t)\right]^{\frac{\sigma_1-\sigma_2-1}{\sigma_2}} \\
&\quad + (1-a)(1-b)^{\frac{1}{\sigma_2}}\left(\frac{\sigma_1-1}{\sigma_2}\right)\left[L_u(t)+P(t)\right]^{\frac{-1}{\sigma_2}}\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_1-1}{\sigma_2-1}}\Bigg] \\
&\quad \times \frac{1}{\sigma_1-1}\left[a(1-b)^{\frac{1-\sigma_1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_1-1}{\sigma_2}}+(1-a)\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_1-1}{\sigma_2-1}}\right]^{\frac{2-\sigma_1}{\sigma_1-1}} \\
&\quad \times \left(\frac{b}{L_s(t)}\right)^{\frac{1}{\sigma_2}}\left((1-c)\frac{L_s(t)}{L_s^T(t)}\right)^{\frac{1}{\sigma_3}} \\
&= \frac{1}{\sigma_2}\Bigg[a(1-b)^{\frac{1-\sigma_1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_1-\sigma_2-1}{\sigma_2}} \\
&\quad + (1-a)(1-b)^{\frac{1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{-1}{\sigma_2}}\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_1-1}{\sigma_2-1}}\Bigg] \\
&\quad \times \left[a(1-b)^{\frac{1-\sigma_1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_1-1}{\sigma_2}}+(1-a)\left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_1-1}{\sigma_2-1}}\right]^{\frac{2-\sigma_1}{\sigma_1-1}} \\
&\quad \times \left(\frac{b}{L_s(t)}\right)^{\frac{1}{\sigma_2}}\left((1-c)\frac{L_s(t)}{L_s^T(t)}\right)^{\frac{1}{\sigma_3}}.
\end{aligned} \tag{A11}
$$

The sign of $\frac{\partial W_s^P(t)}{\partial P(t)}$ is determined by the sign of $\frac{1}{\sigma_2}$, which is positive. Thus, $\frac{\partial W_s^P(t)}{\partial P(t)} > 0$.

**Proof of Lemma 3**: Rearranging (A3) for $K(t)$, gives the equilibrium stock of traditional capital as

$$K(t) = V(t)\left(\frac{a}{1-a}\right)\left(\frac{1}{1-b}\right)^{\frac{\sigma_1}{\sigma_2}}\left(\frac{L_u(t)+P(t)}{V(t)}\right)^{\frac{\sigma_1}{\sigma_2}}. \tag{A12}$$

This can then be written in per capita terms using $k(t) \equiv \frac{K(t)}{L(t)}$, $p(t) \equiv \frac{P(t)}{L(t)}$, $l_u(t) \equiv \frac{L_u(t)}{L(t)}$, and $v(t) \equiv \frac{V(t)}{L(t)}$ as follows:

$$k(t) = \left(\frac{a}{1-a}\right)\left(\frac{1}{1-b}\right)^{\frac{\sigma_1}{\sigma_2}}\left(\frac{l_u(t)+p(t)}{v(t)}\right)^{\frac{\sigma_1}{\sigma_2}}v(t). \tag{A13}$$

The full expression for the composite input, $V(t)$, can be derived by substituting Equation (3) into Equation (2) which gives the following:

$$
\begin{aligned}
V(t) &= \left[b^{\frac{1}{\sigma_2}}L_s(t)^{\frac{\sigma_2-1}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\left[L_u(t)+P(t)\right]^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_2}{\sigma_2-1}} \\
&= \left[b^{\frac{1}{\sigma_2}}\left(l_s(t)L(t)\right)^{\frac{\sigma_2-1}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\left[l_u(t)L(t)+p(t)L(t)\right]^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_2}{\sigma_2-1}} \\
&= L(t)\left[b^{\frac{1}{\sigma_2}}\left[c^{\frac{1}{\sigma_3}}l_s^T(t)^{\frac{\sigma_3-1}{\sigma_3}}+(1-c)^{\frac{1}{\sigma_3}}l_s^P(t)^{\frac{\sigma_3-1}{\sigma_3}}\right]^{\frac{\sigma_3}{\sigma_3-1}\frac{\sigma_2-1}{\sigma_2}}+(1-b)^{\frac{1}{\sigma_2}}\left[l_u(t)+p(t)\right]^{\frac{\sigma_2-1}{\sigma_2}}\right]^{\frac{\sigma_2}{\sigma_2-1}}.
\end{aligned} \tag{A14}
$$

This expression can then be divided by $L(t)$ on both sides to yield an expression for $v(t) \equiv \frac{V(t)}{L(t)}$. This gives the following and completes the proof.

$$v(t) = \left[ b^{\frac{1}{\sigma_2}} \left[ c^{\frac{1}{\sigma_3}} l_s^T(t)^{\frac{\sigma_3-1}{\sigma_3}} + (1-c)^{\frac{1}{\sigma_3}} l_s^P(t)^{\frac{\sigma_3-1}{\sigma_3}} \right]^{\frac{\sigma_3}{\sigma_3-1}\frac{\sigma_2-1}{\sigma_2}} + (1-b)^{\frac{1}{\sigma_2}} [l_u(t)+p(t)]^{\frac{\sigma_2-1}{\sigma_2}} \right]^{\frac{\sigma_2}{\sigma_2-1}}. \quad \text{(A15)}$$

**Proof of Lemma 4**: Given $\mathcal{A}(0) = K(0) + P(0) > 0$ for $t = 0$, the evolution of the economy's total assets in the economy is given by

$$\begin{aligned} \dot{\mathcal{A}}(t) &= \dot{K}(t) + \dot{P}(t) \\ &= s_K(t)I(t) - \delta K(t) + (1 - s_K(t))I(t) - \delta P(t) \\ &= I(t) - \delta(K(t) + P(t)) \\ &= sY(t) - \delta \mathcal{A}(t). \end{aligned} \quad \text{(A16)}$$

Here $s_K(t)$ is the fraction of gross investment in the accumulation of traditional capital and $(1 - s_K(t))$ is the fraction of gross investment in the accumulation of automation capital. Both sides of (A16) can then be divided by $L(t)$ to yield the economy's aggregate capital stock in per capita terms as

$$\frac{\dot{\mathcal{A}}(t)}{L(t)} = \frac{sY(t) - \delta\mathcal{A}(t)}{L(t)} = \frac{sY(t)}{L(t)} - \frac{\delta\mathcal{A}(t)}{L(t)} = sy(t) - \delta\bar{\lambda}(t). \quad \text{(A17)}$$

As $\bar{\lambda}(t) \equiv \frac{\mathcal{A}(t)}{L(t)}$, this implies that

$$\frac{d}{dt}(\bar{\lambda}(t)) = \dot{\bar{\lambda}}(t) = \frac{d}{dt}\left(\frac{\mathcal{A}(t)}{L(t)}\right) = \frac{\dot{\mathcal{A}}(t)L(t) - \dot{L}(t)\mathcal{A}(t)}{(L(t))^2} = \frac{\dot{\mathcal{A}}}{L(t)} - \frac{\dot{L}(t)}{L(t)}\bar{\lambda}(t) = \frac{\dot{\mathcal{A}}(t)}{L(t)} - n\bar{\lambda}(t), \quad \text{(A18)}$$

where $\dot{L}(t) = nL(t)$. Plugging (A17) into (A18) we obtain the following result to complete the proof:

$$\begin{aligned} \dot{\bar{\lambda}}(t) &= \frac{\dot{\mathcal{A}}}{L(t)} - n\bar{\lambda}(t) \\ &= sy(t) - \delta\bar{\lambda}(t) - n\bar{\lambda}(t) \\ &= sy(t) - (\delta + n)\bar{\lambda}(t). \end{aligned} \quad \text{(A19)}$$

**Proof of Proposition 5**: (A13) can be rewritten as the following expression:

$$k(t) = D\big[l_u(t) + p(t)\big]^{\frac{\sigma_1}{\sigma_2}} v(t)^{\frac{\sigma_2-\sigma_1}{\sigma_2}}, \quad \text{(A20)}$$

where $D \equiv \left(\frac{a}{1-a}\right)\left(\frac{1}{1-b}\right)^{\frac{\sigma_1}{\sigma_2}}$, and (A15) can be simplified to the following expression:

$$v(t) = \left[ b^{\frac{1}{\sigma_2}} l_s(t)^{\frac{\sigma_2-1}{\sigma_2}} + (1-b)^{\frac{1}{\sigma_2}} [l_u(t) + p(t)]^{\frac{\sigma_2-1}{\sigma_2}} \right]^{\frac{\sigma_2}{\sigma_2-1}}. \quad \text{(A21)}$$

As we are abstracting from endogenous human capital (i.e., education) decisions, $l_u(t)$, $l_s^P(t)$, $l_s^T(t)$ and $l_s(t)$ are constants. We are interested in a long-run balanced growth path along which the economy grows at a constant rate, despite the absence of technological progress. Along this path of the economy, $p \to \infty$, the following asymptotic approximations hold. From (A21):

$$v(t) \approx (1-b)^{\frac{1}{\sigma_2 - 1}} p(t). \tag{A22}$$

From (A20):

$$\begin{aligned} k(t) &\approx Dp(t)^{\frac{\sigma_1}{\sigma_2}} v(t)^{\frac{\sigma_2-\sigma_1}{\sigma_2}} \\ &\approx Dp(t)^{\frac{\sigma_1}{\sigma_2}} (1-b)^{\frac{\sigma_2-\sigma_1}{\sigma_2(\sigma_2-1)}} p(t)^{\frac{\sigma_2-\sigma_1}{\sigma_2}} \\ &\approx D(1-b)^{\frac{\sigma_2-\sigma_1}{\sigma_2(\sigma_2-1)}} p(t). \end{aligned} \tag{A23}$$

From Equation (14) with (A22) and (A23) substituted in:

$$\begin{aligned} y(t) &\approx \left[ a^{\frac{1}{\sigma_1}} D^{\frac{\sigma_1-1}{\sigma_1}} (1-b)^{\frac{\sigma_2-\sigma_1}{\sigma_2(\sigma_2-1)} \cdot \frac{\sigma_1-1}{\sigma_1}} p(t)^{\frac{\sigma_1-1}{\sigma_1}} + (1-a)^{\frac{1}{\sigma_1}} (1-b)^{\frac{\sigma_1-1}{\sigma_1(\sigma_2-1)}} p(t)^{\frac{\sigma_1-1}{\sigma_1}} \right]^{\frac{\sigma_1}{\sigma_1-1}} \\ &\approx p(t) \left[ a^{\frac{1}{\sigma_1}} D^{\frac{\sigma_1-1}{\sigma_1}} (1-b)^{\frac{(\sigma_2-\sigma_1)(\sigma_1-1)}{\sigma_2(\sigma_2-1)\sigma_1}} + (1-a)^{\frac{1}{\sigma_1}} (1-b)^{\frac{\sigma_1-1}{\sigma_1(\sigma_2-1)}} \right]^{\frac{\sigma_1}{\sigma_1-1}} \\ &\approx p(t) \left[ a(1-a)^{\frac{1-\sigma_1}{\sigma_1}} (1-b)^{\frac{\sigma_2-\sigma_1}{\sigma_1\sigma_2}} + (1-a)^{\frac{1}{\sigma_1}} (1-b)^{\frac{\sigma_1-1}{\sigma_1(\sigma_2-1)}} \right]^{\frac{\sigma_1}{\sigma_1-1}}. \end{aligned} \tag{A24}$$

Note that the expression inside of the square bracket is composed of only constant parameters. Therefore, we shall call this $D_1$:

$$D_1 \equiv \left[ a(1-a)^{\frac{1-\sigma_1}{\sigma_1}} (1-b)^{\frac{\sigma_2-\sigma_1}{\sigma_1\sigma_2}} + (1-a)^{\frac{1}{\sigma_1}} (1-b)^{\frac{\sigma_1-1}{\sigma_1(\sigma_2-1)}} \right]$$

Taking logarithms of each side of (A22) and then taking the time derivative yields the following expression:

$$\frac{\dot{v}(t)}{v(t)} = \frac{\dot{p}(t)}{p(t)}. \tag{A25a}$$

The same procedure can be applied to (A23) and (A24) to deliver the following:

$$\frac{\dot{k}(t)}{k(t)} = \frac{\dot{p}(t)}{p(t)}. \tag{A25b}$$

$$\frac{\dot{y}(t)}{y(t)} = \frac{\dot{p}(t)}{p(t)}. \tag{A25c}$$

To complete the proof, (A25b) and (A25c) can be combined to yield $\frac{\dot{k}(t)}{k(t)} = \frac{\dot{p}(t)}{p(t)} = \frac{\dot{y}(t)}{y(t)}$.

**Proof of Proposition 6**: From Proposition 5 we know $\frac{\dot{k}(t)}{k(t)} = \frac{\dot{p}(t)}{p(t)} = \frac{\dot{y}(t)}{y(t)}$. Let us call this growth rate $g$. Given the assumption that $l_u(t)$ is a constant, the following expression is the time derivative of (A20).

$$\dot{k}(t) = D(1-b)^{\frac{\sigma_2-\sigma_1}{\sigma_2(\sigma_2-1)}} \dot{p}(t). \tag{A26}$$

Recall investment is shared between traditional capital and automation capital, so (A19) implies:

$$\dot{k}(t) + \dot{p}(t) = sy(t) - (\delta + n)(p(t) + k(t)). \tag{A27}$$

Plugging (A26) into (A27) then yields

$$\dot{p}(t)\left[1 + \left(\frac{a}{1-a}\right)(1-b)^{\frac{1-\sigma_1}{\sigma_2-1}}\right] = sy(t) - (\delta + n)(p(t) + k(t)). \tag{A28}$$

We then define $D_2 \equiv 1 + \left(\frac{a}{1-a}\right)(1-b)^{\frac{1-\sigma_1}{\sigma_2-1}}$. Plugging (A23) and (A24) into (A30) then yields

$$\begin{aligned}
\dot{p}(t) &= \frac{1}{D_2}\Big[sD_1 p(t) - (\delta + n)\big[(D_2 - 1)p(t) + p(t)\big]\Big] \\
&= \frac{1}{D_2}\big[sD_1 p(t) - (\delta + n)p(t)D_2\big] \\
&= \frac{1}{D_2}p(t)\big[sD_1 - (\delta + n)D_2\big].
\end{aligned} \tag{A29}$$

From this expression we divide by $p(t)$ to get the growth rate of the automation capital per capita:

$$\frac{\dot{p}(t)}{p(t)} = \frac{1}{D_2}\big[sD_1 - (\delta + n)D_2\big]. \tag{A30}$$

In order to arrive at a solution where the long-run economic growth is positive, it must hold that $\frac{\dot{p}(t)}{p(t)} > 0$ for $p \to \infty$. As $\frac{1}{D_2} > 0$, we only need to look at the asymptotics of $\big[sD_1 - (\delta + n)D_2\big]$. As long as the expression in the squared bracket term is positive, then for $p \to \infty$, $\frac{\dot{p}(t)}{p(t)} > 0$ . Thus, the long-run growth rate of the economy is $g \equiv \frac{\dot{k}(t)}{k(t)} = \frac{\dot{p}(t)}{p(t)} = \frac{\dot{y}(t)}{y(t)} = \frac{sD_1-(\delta+n)D_2}{D_2}$.