Worker Skill Composition, Competition and Innovation

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June 3, 2022

Abstract

I build a model of product market competition and innovation, in which heterogeneously skilled workers perform innovative activities within a firm. I propose a new formula to capture *worker skill composition*, which determines a firm's innovative capacity and reflects an interaction between workers of different skill types in an organisational setting. The model highlights that worker skill composition varies with skill endowment, particularly when firms can benefit from knowledge spillovers. As a result, the skill spread between worker types and profit incentive effects of competition generate an inverted-U shape relationship between worker skill composition and competition.

Keywords— Market competition, innovation, skill composition, knowledge spillover

JEL: J24,L1,O3,O31

1 Introduction

Innovation is an important driver of growth, both at micro level for firms and also at the level of an economy as a whole. Understanding *how* and *why* a firm engages in development of a new product or process continues to be pertinent, particularly given the fast paced nature of the modern technological environment. In this paper, I investigate these questions, by elucidating firm's innovative capabilities, and examining the external forces that may affect its decision to

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innovate. I start from the premise that a firm's measure of performance such as profits and revenue, as well as its organisational practices or culture are in fact a result of an individual or a group of workers' decisions and actions, and study the microfoundations of a firm's innovative performance at individual workers, which, to date, remain underexplored. Process of innovation involves more than strategic decision-making by those in managerial positions, and requires input from others to execute the plans and deliver outcomes. A firm's innovative capability is therefore contingent on the behaviour of the overall workforce engaged in the innovation process.

I build on the theoretical framework of step-by-step innovation and competition (Aghion, Harris, Howitt, & Vickers, 2001; Aghion, Bloom, Blundell, Griffith, & Howitt, 2005), and model a firm's decision to invest in research and development (R&D) as a function of the skill endowment and extent of product market competition. I define a firm's investment in R&D as a measure of its innovative capacity, which depends on the composition of its workers with heterogeneous skills in performing innovative activities. The success rate of a firm's R&D endeavours depends on the skill composition of its workers, and a firm seeks to innovate to maximise the potential incremental postinnovation rents, given a level of competition in the market.

The model's mechanism can be characterised by the following main features: (i) workers are either high-skilled or low-skilled in performing innovative activities, (ii) a firm's worker skill composition is a product of the interaction between workers of different types and influences the success rate of a firm's innovation, and (iii) the extent of low-skilled workers' ability to absorb knowledge spillovers, together with product market competition, impact the need for high-skilled workers. Crucially, the skill spread between high- and low-skilled workers affects the innovative capacity of a firm, because if low-skilled workers are not able to fully internalise the knowledge spillovers, more high-skilled workers are required to compensate this. If the level of competition is such that the expected postinnovation rents are higher than the current profits, firms decide to invest in R&D. An optimal R&D investment ensures a sufficient level of innovative capacity, that is, the skill composition among workers, to successfully implement innovation.

My analysis reveals that the *escape-competition* and *Schumpeterian* effects of competition on innovation are echoed on skill composition in firms. If a change in product market competition results in potential incremental postinnovation profits, then it induces firms to increase their innovative capacity by investing in more high-skilled workers to engage in R&D. When competition is high and firms are technologically on par, there is a greater incentive to capture the market by introducing a new product. Subsequently, firms invest in improving their innovative capacity to increase the chance of succeeding in innovation, which is the escape-competition effect. On the other hand, when competition is high and a firm is a technological follower, there is less incentive for it to invest in improving their innovative capacity as further increases in competition imply there is not much benefit in catching up, leading to the Schumpeterian effect where innovation decreases. The inverted-U shape relationship between skill composition and competition reflects firms' investment levels and their technological state in relation to that of their rivals.

The analysis raises implications on demand for skill types at different levels of market competition. The non-linear relationship between competition and skill composition implies that market competition at either extremes may exacerbate a low-skill concentrated workforce. In other words, when firms do not foresee innovation as being economically incentivising due to the nature of the competitive environment, firms are discouraged from improving their capabilities. Moreover, it answers questions on skill diversity in innovation. If the skill endowment of low-skilled workers is significantly lower compared to high-skilled workers, that is, there is a greater level of diversity across skill types, then firms need more high-skilled workers to perform innovative activities successfully.

The rest of the paper is organised as follows. In Section 2, I review existing literature related to this study. Section 3 outlines the model and derives predictions based on comparative statics, and Section 4 concludes. All proofs are relegated to Appendix A.

2 Related literature

The paper relates to several different strands of existing literature. First, the framework utilises the endogenous technological growth theory (Grossman & Helpman, 1991a, 1991b; Aghion & Howitt, 1998), and builds on the model of innovation and competition by Aghion, Harris, and Vickers (1997), Aghion et al. (2001) and Aghion et al. (2005). I contribute by incorporating heterogeneous worker skills within the firm innovation process, which often remains a 'black box' in traditional endogenous growth theory with general equilibrium models analysed at aggregated levels (Aghion & Tirole, 1994). In particular, I present new results on the relationship between competition and innovative capabilities at firm level. Notably, by attributing R&D expenditure as an investment to improve its worker skill composition, I unfold the elements of the R&D intensity analysed in the existing literature on step-by-step innovation and competition (Aghion et al., 2005).

The central feature of my model is the contribution of workers to a firm's success rate of innovation. The idea relates to a number of papers emphasising the role of human capital and skilled labour in technological change, including Nelson and Phelps (1966), Acemoglu (2002a, 2002b), Acemoglu, Aghion, and Zilibotti (2006), Acemoglu and Autor (2011), and Acemoglu, Gancia, and Zilibotti (2012). The main hypothesis is that more educated individuals, or skilled workers are required for creation of new knowledge. The presence of knowledge spillovers enables imitation, or standardisation which is relatively less complex than innovation. Therefore, having high skills is more important for success in innovation than adoption. My approach differs from these models in that I model innovation as a process that involves creation and implementation of new knowledge within a firm environment, and posit a particular functional form of aggregating different worker skill types to reflect firm-level innovative capacity.

The second category of existing literature related to this study is on the influence of firm-level capability or stock of knowledge on innovation, including the dynamic capabilities framework (Teece, Pisano, & Shuen, 1997; Teece, 2007, 2010, 2016) and absorptive capacity (Cohen & Levinthal, 1989, 1990; Aghion, Jaravel, Cohen, & Levinthal, 2015). The main difference between the innovative capacity that is captured by worker skill composition in this study and the dynamic capabilities framework is that the latter focuses on the role of managers for building knowledge at the firm level. The underlying premise of the formulation of worker skill composition is that a high-skilled worker (manager) can contribute to her full potential if her coworkers are also high-skilled. Furthermore, my modelling approach embeds the notion of absorptive capacity as a necessary driver for innovation. Cohen and Levinthal (1990) argue that the firm-level ability to exploit, assimilate and commercialise new information is an output of various factors, including individual workers' absorptive capacity and communication across the organisation. Therefore, the proposed formula for skill composition at firm level in this paper succinctly captures both the capability of a firm's overall workforce engaging in innovation and the complexity of an organisational environment as an aggregation of heterogeneous individuals.

Third, the concept of interaction between worker types and its consequences on firm's ability to innovate is linked to the organisational economics literature on corporate culture focusing on the role of shared knowledge on firm performance and growth. Li (2016) shows that a lack of shared knowledge results in coordination failure and knowledge fragmentation within an organisation, and that a rapid growth strategy without having established shared knowledge may be detrimental to the firm. Crémer (1993) defines corporate culture as the stock of knowledge that is shared by the members of an organisation but not by the general public, and it facilitates a more efficient way of responding to information received from external sources at an organisational level. The theoretical concepts in both of these papers are plausible explanations for modelling worker skill composition as an interaction between worker types in this paper.

Finally, the paper also relates to literature on diversity and technological progress. Acemoglu (2012) presents a model in which diversity between individual researchers' expertise, beliefs and knowledge results in non-profit maximising behaviour from researchers, which translates into diversity in the types of technology being developed. For a social planner, this is a better outcome compared to when there is no diversity across researchers as having a variety of types of innovation benefits the society. Related to this is the model of heterogeneous problem solving agents by Hong and Page (2001). In their model, agents have differentiated perspectives in their approach to problems, but are still homogeneous in their value function of the solution with no communication costs. The authors show that diversity is optimal, because it increases the set of possible solutions that agents evaluate. This paper focuses on diversity in terms of skills required for innovation in a single organisational setting, and offers contrasting implications, by showing that absent sufficient profit incentives, increased heterogeneity in worker skills can impede firm innovation.

3 Theoretical framework

3.1 The step-by-step innovation model

The basic setup

The theoretical framework is based on the endogenous growth models of step-by-step innovation and competition (Aghion & Howitt, 1998; Aghion et al., 2001; Aghion et al., 2005). In this subsection, I outline the main features of these models and their predictions on the relationship between competition and innovation.

There exist two firms $i = 1, 2$ in an industry v. Each firm is a duopolist, producing output y_i according to a constant returns to scale production function, taking labour as the only input.

Firms can improve their technology by investing in R&D, and a successful innovation results in a reduction in the cost of production. Each firm's output per worker, A_i, is equal to

$$A_i = \gamma^{k_i},$$

where k_i denotes the technology level of firm i, and $\gamma > 1$ is a parameter of the size of a leading-edge innovation. Given A_i, the unit cost of production remains the same regardless of the scale of production, and firm i's unit cost is equal to γ^{-k_i} units of labour. Hence, an industry v can be fully characterised by a pair of integers, (k_v, m_v) where k_v is the leader's technology, and m_v is the technological gap between the industry leader and follower.

Crucially, innovation occurs in a step-by-step process, in which a laggard firm must catch up with the leader before it can become a leader itself. Furthermore, assume that the maximum sustainable technological gap is $m_v = 1$. Specifically, when a firm that is already ahead of the other firm innovates further, the laggard *automatically* adopts the leader's previous technology, so that the technological gap between the two firms remains at one step. At any point in time, an industry then consists of: (i) *levelled* or *neck-and-neck* state, where both firms are technologically even, and (ii) *unlevelled* state, where one firm (the *leader*) has a more advanced technology than its competitor (the *laggard* or *follower*).

By spending $\psi(n) = n^2/2$ in units of labour, a leader or a neck-and-neck firm moves one step ahead with a Poisson hazard rate $n \in [0, 1]$. Let n denote the "innovation rate" or "R&D intensity" of the firm. A follower firm can copy a leader's technology without spending any R&D cost at a hazard rate h, where $h \geq 0$ is a parameter that measures R&D spillovers or ease of imitation. Hence, a follower firm can move one step ahead to draw level with the leader by spending R&D cost $\psi = n^2/2$, with a Poisson hazard rate $(n + h) \in [0, 1]$.

Define π_m to be the equilibrium profit flow of a firm at state $m \in \{-1, 0, 1\}$. Competition in the market is modelled by the inverse of the degree to which the two firms in a levelled state can collude. They do not collude if the industry is unlevelled, which implies that the laggard firm makes zero profit. The leader's profit is equal to the difference between its revenue, normalised to 1, and the cost. The profits of the follower and leader in an unlevelled industry are respectively:

$$\pi_{-1} = 0 \qquad \text{and} \qquad \pi_1 = 1 - \gamma^{-1}.$$

If firms are unable to collude in a levelled state, they enter a Bertrand competition and both earn a zero profit. Assume that with maximum collusion, the combined profit of the two firms is equivalent to a technological leader's profit. More generally,

$$\pi_0 = \varepsilon \pi_1, \qquad 0 \leq \varepsilon \leq \frac{1}{2},$$

and define Δ as the parameter measuring product market competition. Then product market competition is equal to $\Delta = 1 - \varepsilon$, that is, the fraction of a leader's profits that a neck-and-neck firm can earn through collusion.

The *escape-competition* and *Schumpeterian* effects

The R&D intensity rate, n_0 or n_{-1} of neck-and-neck and laggard firms respectively, varies with the measure of competition, Δ. It produces two opposing effects depending on the firm's technological state, because the anticipated effect of competition is different for a neck-and-neck compared to a laggard firm. The basic model predicts that n_0 increases with higher competition because greater competition negatively affects the preinnovation rents, and therefore firms engage in more innovation, which is the *escape-competition* effect. On the other hand, n_{-1} decreases with higher competition, because the postinnovation rents for a laggard firm is decreasing in competition, thus investing in R&D becomes less attractive.

The total effect of competition on innovation, therefore, depends on the state of a sector. In an unlevelled state, the Schumpeterian effect takes place, while in a levelled state, the escape-competition takes place. Moreover, the model implies the transition, or the flow, from levelled to unlevelled state will be quicker, because the R&D intensity of neck-and-neck firms increases with higher competition. On the other hand, due to the Schumpeterian effect, a sector may remain unlevelled for a prolonged period of time. On an aggregate level, the expected technological gap is increasing in the degree of competition, as sectors remain in unlevelled state and the fraction of unlevelled sectors increases. The higher the degree of competition, the smaller the share of neck-and-neck industries in the economy. Thus, the model further predicts that in industries with a higher level of knowledge spillovers, h, the escape-competition effect will be stronger in that on average, firms are more likely to be neck-and-neck, because innovation requires less effort in such industries.

3.2 Opening the blackbox of firm R&D intensity

I build on the model of step-by-step innovation and competition to investigate the microfoundations of a firm's decision on R&D intensity, n, in response to the product market competition and expected postinnovation rents. In the model à la Aghion et al. (2005), the success rate of innovation depends on a Poisson rate n, which implies that the more units of labour a firm employs in R&D, the higher the probability of success. I extend the framework by considering the type of workers a firm employs to increase their chance of succeeding in innovation. Central to the extended framework are workers with heterogeneous skill endowment in performing innovative activities and the nature of interaction between workers, which directly affect the probability of successful innovation. In other words, I interpret a higher n as the firm's investment in ensuring a more high-skilled team of workers to engage in innovation.

I describe the specifics of the extended framework in the rest of the subsection. I begin by discussing the workers and their skills, and characterise the interplay between workers of different types within a firm environment. I then outline the innovation process, incorporating the innovation skill levels.

Workers

An economy is populated with a continuum of unit mass individuals indexed by $j \in [0, 1]$, supplying labour inelastically. All workers are equally productive in production activities, but have heterogeneous *innovation skills*. Each worker is endowed with either low or high innovation skills, denoted by $s_j \in \{\underline{s}, \overline{s}\}$. A worker's innovation skills remain constant over time, and takes a value between $(0, 1]$, that is, $0 < \underline{s} < \overline{s} \leq 1$. I assume that the compensation structure reflects a worker's contribution to both production and innovation, in other words, the incentive constraints of workers choosing between production or innovation are met.

Workers of high and low innovation skill types both participate in an innovation process. Innovation skills are abilities that individuals possess, that enable them to identify opportunities, evaluate benefits of potential innovation and execute them. In practice, these are a combination of cognitive, technical and behavioural worker characteristics that are difficult to measure, but sought after by organisations in their employees. It incorporates the notion of absorptive capacity at an individual worker level, that is, the ability to process and assimilate knowledge from external sources and apply appropriately to their own needs.

Innovation and worker interaction

An innovation entail introducing a new product or technology and becoming a technological leader, or climbing the existing technological step by adopting the newest technology to become levelled with the rival. Engaging in a successful innovation process at time t results in $k_{i(t+1)} = k_{it} + 1$. The probability that a firm succeeds in innovation is determined by the combined innovation skills of workers involved in the process, which I refer to as the *worker skill composition*, S. In relation to the original model of Aghion et al. (2005), the variable n, which is the R&D intensity, is now represented by the worker skill composition S, that is, $\psi(n) = n^2/2 = S^2/2$.

I now define S in each firm in terms of the skill level of the workforce engaging in innovation. The worker skill composition for firm i at time t has the below specific functional form:

$$S_{it} \equiv \overline{s}^{\alpha_{it}} \underline{s}^{(1-\alpha_{it})}, \tag{1}$$

where α_{it} is the share of high-skilled workers. The functional form of S_{it} captures the interaction where, at any given levels of skill endowment for each type, the mechanism through which S_{it} increases is by increasing α_{it}. Firms must now determine the R&D intensity by solving for the optimal share of high-skilled workers, given the skill endowment of both worker types, its technological state and competitive environment.

Crucially, I assume that the composition of workers matters for the interaction between the two worker types, not just the absolute difference between the high- and low-skilled workers. The contribution of each worker to the likelihood of success is not additive, but is a product of combining skills of all workers. To put simply, consider a firm with a high-skilled worker whose coworkers are all low-skilled. Then, the high-skilled worker does not contribute markedly to the overall organisational skill level. The contribution of the high-skilled worker increases once the firm hires another high-skilled worker. The two high-skilled workers complement each other and augment the organisational skill.

A possible interpretation of worker skill composition is the potential to bring value to an organisation through performing innovative activities, and a high-skilled worker's potential can only be realised if there are other workers who also share these attributes. Firms engaging in an innovation process, therefore, invest to construct a level of overall innovation capacity required for developing a new technology through the composition of workers, by altering the share of high-skilled workers within the workforce. S_{it} then represents the "average" innovation skill level of a firm's workers, and signifies a measure of firm-level knowledge which embodies a type of complexity of knowledge flow in an organisational environment.

It may be argued that a firm with a larger headcount of high-skilled workers than a smaller firm has a higher "volume" of knowledge required for developing a new product, and therefore is more likely to succeed in innovation. In the current framework, however, I postulate that innovation requires all workers involved in the innovation process communicate the knowledge with each other to develop and implement the technology, and the extent of such knowledge diffusion depends on the relative "share" of each skill type to one another. This implies that regardless of the size of the total number of workers engaging in innovative activities, a firm chooses the relative share of each worker type to maximise the probability of success.

Naturally, the highest organisational level of innovation skills is achieved when all workers participating in an innovation process are of high-skill type, however given the assumptions that worker compensation reflects their contribution, and that the probability of success in innovation depends on the firm's technological state compared to its rival, this is not always optimal.

Technological progress

Firm i's technology k at $(t+1)$ can be expressed generally as:

$$k_{i(t+1)} = f(S_{it}). \tag{2}$$

Figure 1 summarises the transition between technological states and the corresponding probability of successful innovation. For each firm, I assume that the number of workers engaging in innovation is proportional to total output y_{it}, and is normalised to 1. In an unlevelled state, a leader firm spends R&D cost $\psi_1(\overline{s}, \underline{s}, \alpha_1) = \frac{(\overline{s}^{\alpha_1} \underline{s}^{(1-\alpha_1)})^2}{2}$ and moves one technological step ahead, with success rate $p_1 = \overline{s}^{\alpha_1} \underline{s}^{(1-\alpha_1)}$. A follower firm can always successfully copy the leader's previous technology and move one step ahead without spending any R&D cost and maintain the one step gap. In other words, the success rate of copying does not depend on worker skill composition. Then, $\frac{(\overline{s}^{\alpha_{-1}} \underline{s}^{(1-\alpha_{-1})})^2}{2}$ is the R&D cost of a follower firm innovating with success rate $(\overline{s}^{\alpha_{-1}} \underline{s}^{(1-\alpha_{-1})} + \underline{s})$ to draw level with its rival. The higher probability of success for a follower firm reflects the fact that there is more advanced technology that can be observed in the market, which I assume can be done by workers of any type. The $\underline{s}$ is analogous to the follower's hazard rate h in the original

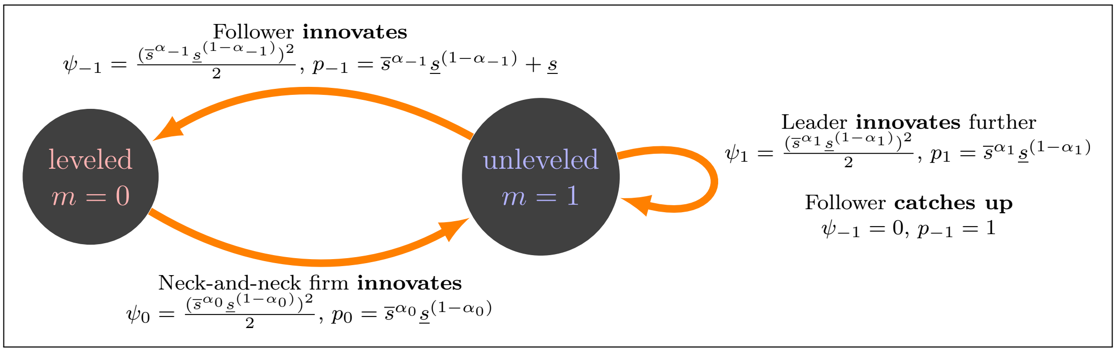

Figure 1: Transition diagram of the stationary Markov process

model by Aghion et al. (2005). I adopt the view that institutional protection such as intellectual property rights plays a role of making knowledge free to the public (Acemoglu, 2012), and the extent of utilising this existing knowledge is dependent on low-skill endowment, which can be thought of as the absorptive capacity of low-skilled workers. The automatic catch-up assumption, however, implies that the leader does not benefit from further innovation, so in an unlevelled state, $\alpha_1 \to 0$ as fewer innovative tasks requiring high skills are performed. In a levelled state, by spending the R&D cost $\psi_0(\bar{s}, \underline{s}, a_0) = \frac{(\bar{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)})^2}{2}$, a neck-and-neck firm moves one technological step ahead with success rate $p_0 = \bar{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)}$.

I model competition in the same way as Aghion et al. (2005). The comparative statics of main interest are the signs of $\frac{\partial\alpha}{\partial\bar{s}}$, $\frac{\partial\alpha}{\partial\underline{s}}$ and $\frac{\partial\alpha}{\partial\Delta}$, that is, the effects of skill endowment and market competition on firms' share of high-skilled workers, which determines the success rate of innovation.

3.3 Equilibrium share of high-skilled workers

I describe the steady state equilibrium share of high-skilled workers by solving for a symmetric stationary Markov equilibrium.

Proposition 1. *The equilibrium share of high-skilled workers of each neck-and-neck firm is*

$$\alpha_0 = \frac{\ln\left(\sqrt{\underline{s}^2 + 2\Delta\pi_1} - \underline{s}\right) - \ln(\underline{s})}{\ln(\bar{s}) - \ln(\underline{s})},$$

and the equilibrium share of high-skilled workers of a follower firm is

$$\alpha_{-1} = \frac{\ln\left(\sqrt{\underline{s}^2 + (\sqrt{\underline{s}^2 + 2\Delta\pi_1} - \underline{s})^2 + 2\pi_1} - \sqrt{\underline{s}^2 + 2\Delta\pi_1}\right) - \ln(\underline{s})}{\ln(\bar{s}) - \ln(\underline{s})}.$$

Both α_0 and α_{-1} decrease with higher skill endowment of each type. In particular, α_0 decreases with higher values of $\underline{s}$ at a faster rate than α_{-1}.

To understand these results, note that the equilibrium overall worker skill composition for each neck-and-neck (S_0) and follower (S_{-1}) firm is a function of its ability to utilise knowledge spillover, which, by assumption, depends on the skill endowment of low-skilled workers $\underline{s}$, and the potential postinnovation rents expressed in terms of competition and a leader's profits, $\Delta\pi_1$. The case when the high-skill endowment increases is intuitive. Given the product market competition and endowment of the low-skill type remain constant, an increase in the skill endowment of high-skilled workers implies that it now takes fewer high-skilled workers to achieve the same level of overall worker skill composition than before, therefore, the share of high-skilled workers decreases.

The interpretation of the result in which α_0 decreases in higher low-skill endowment, $\underline{s}$, is less straightforward. Initially, an increase in low-skill endowment results in a decrease in high-skilled workers as low-skilled workers can now contribute more to the overall worker skill composition. In fact, an increase in the productivity of either type of workers implies that there can now be fewer high-skilled workers. However, importantly, an increase in low-skill endowment has further implications as there is now a higher probability that a rival firm benefits from the knowledge spillover, and in the case of a neck-and-neck firm, this acts as a disincentive to innovating and becoming a leader. Therefore, a greater low-skill endowment creates a further discouraging effect on a neck-and-neck firm's expected postinnovation rents, and innovation maybe slow in this case.

For a follower firm, the general effect of replacing an increase in productivity with fewer high-skilled workers (that is, lower α_{-1}) would take place, however, in contrast to a neck-and-neck firm, a larger $\underline{s}$ indicates a higher chance of catching up to become neck-and-neck with the rival. Therefore, a higher low-skill endowment implies that there are now fewer high-skilled workers needed to achieve the same level of worker skill composition, but also a higher probability of absorbing knowledge spillover from the current technological leader. This results in a smaller decrease in the share of high-skilled workers for a follower firm, compared to the effect of higher $\underline{s}$ on a neck-and-neck firm's share of high-skilled workers.

The effects of an increase in both skill types described above can be interpreted in the opposite direction, that is, lower endowment of either skill type increases the share of high-skilled workers in equilibrium. Furthermore, a lower $\underline{s}$ is more detrimental to a follower firm than a neck-and-neck firm, that is, the share of high-skilled workers increases at a faster rate for a follower firm than for a neck-and-neck firm, to compensate for the decline in the low-skilled workers' absorptive capacity. For clarity, define the difference between high- and low-skill endowment as the *skill spread* between workers. An increase in skill spread then refers to either: (i) an increase in $\overline{s}$, labelled as *high-skill driven*, or (ii) a decrease in $\underline{s}$, labelled as the *low-skill driven* increase, expansion or widening in skill spread. The equilibrium share of high-skilled workers in each neck-and-neck and follower firm decreases with high-skill driven widening in skill spread, while it increases with low-skill driven widening in skill spread. In particular, in the latter case, a follower's equilibrium share of high-skilled workers increases at a faster rate than that of a neck-and-neck firm.

A firm's decision to increase the share of high-skilled workers thus depends on the skill endowment and essentially, on the competitive environment. I proceed to describe the effect of competition on the equilibrium share of high-skilled workers, and explain a firm's choice of skills in terms of competition and expected postinnovation rents. The results refer to the Schumpeterian and escape-competition effects described in the basic step-by-step innovation model in Section 3, and provide a reinterpretation of the relationship between competition and innovation suggested by Aghion et al. (2005).

Proposition 2. *The equilibrium share of high-skilled workers for a neck-and-neck firm increases in product market competition, while for a follower firm, it decreases with higher product market competition.*

The effect of competition on firms' worker skill composition depends on whether a market is in a levelled or unlevelled state. For a follower firm, the rents it may capture from becoming levelled with the leader reduce as competition increases. This translates to a decrease in the share of high-skilled workers, as there is less incentive to invest in R&D, which is the *Schumpeterian* effect. This results in a lower overall worker skill composition for a follower firm. In a levelled state, the *escape-competition* effect takes place, as the incremental value of becoming a technological leader increases with higher product market competition. Therefore, the share of high-skilled workers increases as neck-and-neck firms invest more in R&D.

Next, following Aghion et al. (2005), I derive under which conditions the escape-competition and Schumpeterian effects of product market competition dominate. In the current framework, the pattern of aggregate worker skill composition follows that of aggregate innovation flow in the model by Aghion et al. (2005). The share of high-skilled workers is a determining factor of innovation, hence the aggregate worker skill composition and innovation move in parallel. In other words, the larger the number of firms innovating, the greater the share of high-skilled workers, therefore the higher the firm innovative capacity represented by worker skill composition. The overall effect of competition on worker skill composition will depend on the fraction of levelled versus unlevelled sectors in the steady-state, which is dependent on equilibrium shares of high-skilled workers in both types of sectors. Let μ_1 (μ_0) denote the steady-state probability of a market being in an unlevelled (levelled) technological state. The steady-state probability that a market moves from being unlevelled to levelled is now a function of worker skill composition and can be expressed as $\mu_1(\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})}+\underline{s})$, and the steady-state probability that a market moves from being levelled to unlevelled is $2\mu_0\overline{s}^{a_0}\underline{s}^{(1-\alpha_0)}$. In steady state, these two probabilities must be equal:

$$\mu_1(\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})}+\underline{s}) = 2\mu_0\overline{s}^{a_0}\underline{s}^{(1-\alpha_0)}. \tag{3}$$

As a market is either levelled or unlevelled, $\mu_1+\mu_0=1$. Then, the aggregate level of worker skill composition, H, determined by the share of high-skilled workers, α_m, is:

$$\begin{aligned} H = 2\mu_0\overline{s}^{a_0}\underline{s}^{(1-\alpha_0)} + \mu_1(\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})}+\underline{s}) &= 2\mu_1(\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})}+\underline{s}) \\ &= \frac{4\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)}(\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})}+\underline{s})}{2\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)}+\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})}+\underline{s}}. \end{aligned} \tag{4}$$

The next prediction is linked to the findings of Aghion et al. (2005) on the aggregate innovation rate, and characterises worker skill composition as an extensive feature of innovation. From Proposition 2, α_0 is an increasing function of Δ, and therefore $\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)}$ is increasing in market competition. Hence, I use $S_0=\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)}$ as a proxy for product market competition. Given $\frac{1}{2}\leq\Delta\leq 1$, S_0 takes values on the interval $[S_0^{\min}=(\sqrt{\underline{s}^2+\pi_1}-\underline{s}), S_0^{\max}=(\sqrt{\underline{s}^2+2\pi_1}-\underline{s})]$, with $S_0^{\min}$ and $S_0^{\max}$ corresponding to the minimum competition where firms fully collude, and maximum competition with no collusion, respectively.

Proposition 3. *The aggregate worker skill composition $v(S_0)$ follows an inverted-U pattern whenever the value $\tilde{S}_0=\sqrt{(\underline{s}^2+2\pi_1)/3}$ is interior to the interval $[S_0^{min}, S_0^{max}]$: it increases with competition S_0 for all $S_0\in[S_0^{min},\tilde{S}_0)$ and decreases for all $S_0\in(\tilde{S}_0, S_0^{max}]$. If $\tilde{S}_0>S_0^{max}$, then the aggregate worker skill composition increases with S_0 for all $S_0\in[S_0^{min}, S_0^{max}]$ so that the escape-competition effect dominates. If $\tilde{S}_0<S_0^{min}$, then the aggregate worker skill composition decreases with S_0 for all $S_0\in[S_0^{min}, S_0^{max}]$ so that the Schumpeterian effect always dominates.*

The inverted-U shape of aggregate worker skill composition can be explained by the firms' incentives to innovate, which depend on product market competition and technological state of the industry. Consider an industry starting with a low level of competition. If firms are technologically neck-and-neck, both firms are generating a profit that is closer to a technological leader's profit, therefore they have no further incentive to innovate. In this case, increasing worker skill composition becomes superfluous, and it will be much slower for the industry to leave the levelled state. The aggregate worker skill composition in the industry will be highest if the industry were unlevelled as firms are more incentivised to innovate, and the unlevelled state will quickly become levelled, leading to a dominance of the *escape-competition* effect.

Conversely, consider an industry with a high level of competition initially. If firms are technologically neck-and-neck, the incremental profit from becoming a technological leader is much larger, therefore firms have higher incentives to increase the share of high-skilled workers in order to succeed in innovation. On the other hand, if the industry is in an unlevelled state, the follower has relatively low incentives to innovate, because the prospect of earning incremental profits even when it becomes a leader is relatively low, due to high competition. Thus the industry will be slow to leave the state, leading to a dominance of the *Schumpeterian* effect. In this case, an increase in competition will only lower the follower's incentives to innovate and the worker skill composition remains low, further prolonging the unlevelled state. Therefore, in the steady state, the higher the degree of competition, the smaller the fraction of neck-and-neck sectors on the whole.

3.4 Technological gap, skill spread and competition

The model further extends the predictions on the expected technological gap by Aghion et al. (2005), who show that the expected technological gap is increasing in product market competition. Recall from Equation (2) the technology is a function of worker skill composition. The technological gap between a leader and follower is therefore a function of the difference between the worker skill composition of the two firms.

Proposition 4. *The expected technological gap increases in low-skill driven skill spread.*

As previously discussed in Proposition 2, if an industry is in a neck-and-neck state, the higher the product market competition, the higher the incentives to innovate, and therefore the higher the share of high-skilled workers, and the industry will quickly become unlevelled again. If an industry is in an unlevelled state, the higher the product market competition, the lower the incentives to innovate, resulting in a prolonged unlevelled state. Moreover, compared to a neck-and-neck firm, the share of equilibrium high-skilled workers increases in low-skill driven skill spread at a faster rate to maximise the probability of success in innovation. This stems from the fact that, as the low-skilled workers' absorptive capacity decreases, more high-skilled workers are needed to make up for it, which further discourages innovation. This leads to the intuition is that industries with a wider skill spread driven by lower $\underline{s}$ will see a larger gap between worker skill composition over time, compared to industries with a higher level of $\underline{s}$.

The next prediction formulates that there is a positive interaction between the *escape-competition* effect and low-skill endowment on the aggregate worker skill composition in the industry. The intuition is that in industries with higher competition and a smaller skill spread driven by higher $\underline{s}$, firms innovate more to capitalise on postinnovation rents, resulting in a steeper increase to the peak of the inverted U.

Proposition 5. *The peak of the inverted U is wider and occurs at a higher degree of competition, in more neck-and-neck industries with a smaller skill spread driven by higher $\underline{s}$. Let $\tilde{\Delta}$ be the incremental profit at which $S_0 = \tilde{S}_0 = \sqrt{(\underline{s}^2 + 2\pi_1)/3}$. Both $\tilde{\Delta}$ and $v(\tilde{S}_0)$ are increasing in $\underline{s}$.*

Suppose competition is high and firms are generating a low profit, and the skill spread in an industry is small, that is, low-skilled workers share many of the attributes of high-skilled workers. Firms are more likely to engage in innovation for two reasons: (i) the prospect of becoming a leader and winning over the market offsets the discouraging effect of a higher likelihood of the rival catching up, (ii) once a rival innovates, the follower can catch up quickly as it has a high absorptive capacity (high $\underline{s}$). The second point leads to the industry reverting quickly back to a levelled state even when a firm innovates, thus there will be more innovation occurring over time in this industry.

If the skill spread in an industry was large, the follower firm's probability of success in innovation decreases, requiring higher a R&D investment and reducing the expected postinnovation rents. Figure 2 shows numerical examples in which $\overline{s} = 0.3$, $\underline{s} = \{0.04, 0.05\}$, $\gamma = 1.02$, and $\pi_1 = 0.02$. A neck-and-neck firm's profit is a fraction of a leader's profit, depending on the degree of competition.[1] Initially, the aggregate worker skill composition increases with higher competition

[1]Recall, $\pi_0 = (1 - \Delta)\pi_i$, where $\frac{1}{2} \leq \Delta \leq 1$.

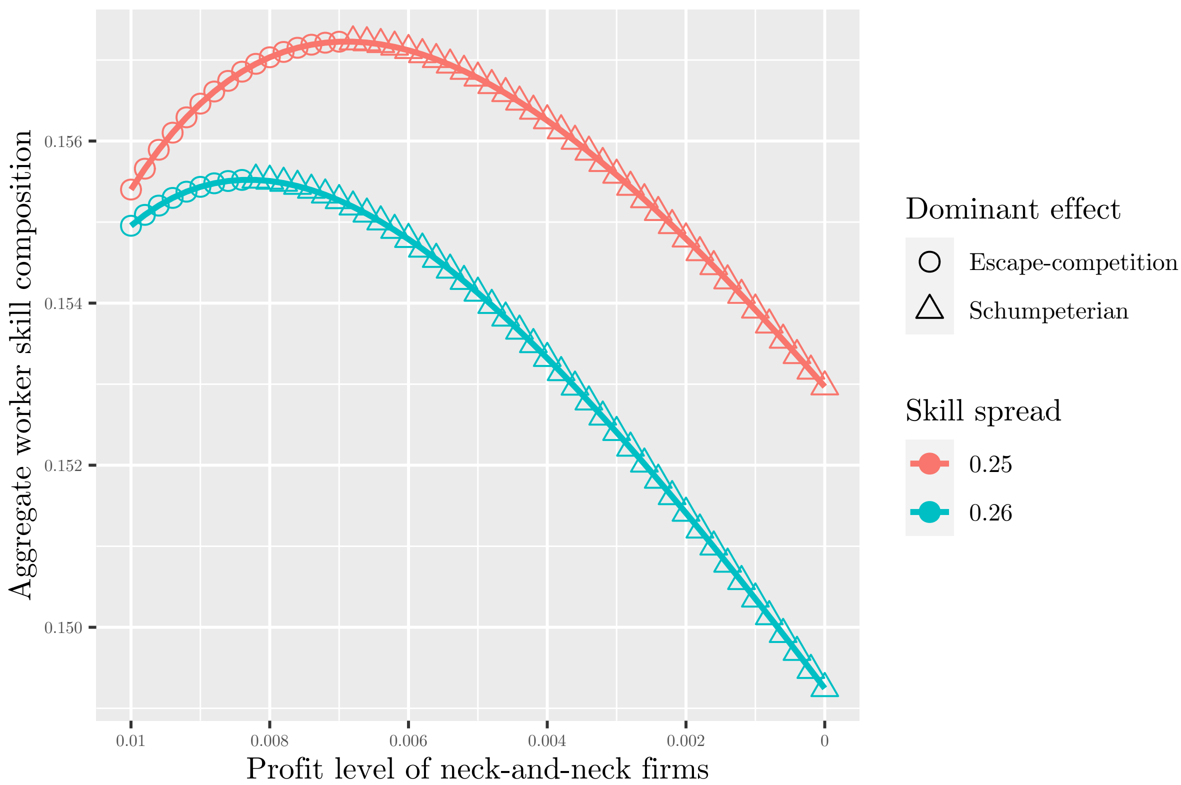

Figure 2: The firm innovative capacity measured by worker skill composition is always higher with greater low-skill endowment. Moreover, the peak of the curve occurs at a higher degree of competition when the skill spread is smaller as a result of higher low-skill endowment.

as more firms innovate to escape competition, and postinnovation rents for a follower (profit level of neck-and-neck firm) are still sufficiently high to incentivise innovation. As competition increases further, the expected postinnovation rents for a follower decreases and eventually the aggregate worker skill composition decreases. As shown by the two curves, the effect of escape-competition is larger and occurs at higher degree of competition when the skill spread is smaller.

4 Conclusion

Workers in organisations often perform multiple tasks. Some of these tasks constitute day-to-day activities, while others might be ad hoc and focus on creating new products and processes when opportunities arise. Successful innovation requires an exhaustive understanding of the current business operations, and to build this knowledge, firms often allocate workers from different teams across the organisation to participate in innovative activities. In some small businesses, the entire workforce may be involved in both operational and innovative activities. In this study, I propose a formula to measure worker skill composition as an indicator of firm-level innovative capacity, taking into account the nature of the interaction between a diverse group of workers. I incorporate the concept of worker skill composition into a model of competition and innovation. Doing so, a firm's worker skill composition becomes an endogenous response to profit incentives and the skill-endowment of workers.

This study reveals new results on how firms might design their workforce in response to the competitive environment. First, the skill endowment of low-skilled workers matters more for a technological follower than the leader, because a follower firm is more likely to engage in innovation if low-skilled workers can absorb more knowledge spillovers. Second, the expected postinnovation rents driven by competition generate an inverted U-shape relationship between competition and worker skill composition. Third, related to the first point made, the technological gap in an industry decreases with smaller skill spread driven by low-skilled workers, as more firms innovate with higher low-skill endowment. Lastly, putting the previous points together, the worker skill composition is highest when competition is high and skill spread is small.

The framework developed in this paper provides an empirically testable model to investigate the above predictions. In particular, the model suggests that either a lack of or too much competition may lower demand for high-skilled workers. Linked employer-employee data from administrative data sources can be used to construct firm-level worker skill composition and obtain information related to firm innovation. Furthermore, the skill spread might vary by industries, and comparing the relationship between worker skill composition and competition in various settings is an interesting direction for future investigation.

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A Appendix

A.1 Proof of Proposition 1

I use Bellman equations to solve for the equilibrium values of the variable of interest, which is the share of high-skilled workers of neck-and-neck and follower firms, α_0 and α_{-1}. Let V_m denote the steady state value of currently being at state $m \in \{-1, 0, 1\}$, that is, the technological gap of two firms in the market, and let r denote the rate of time discount. The Bellman equations are:

$$rV_1 = \pi_1 + (\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})} + \underline{s})(V_0 - V_1), \tag{A.1}$$

$$rV_{-1} = \pi_{-1} + (\overline{s}^{\alpha-1}\underline{s}^{(1-\alpha_{-1})} + \underline{s})(V_0 - V_{-1}) - \frac{(\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})})^2}{2}, \tag{A.2}$$

$$rV_0 = \pi_0 + (\overline{s}^{\hat{\alpha}_0}\underline{s}^{(1-\hat{\alpha}_0)})(V_1 - V_0) - (\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)})(V_0 - V_{-1}) - \frac{(\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)})^2}{2}. \tag{A.3}$$

Equation (A.1) shows that the expected payoff of currently being a technological leader in a market equals the current profit flow π_1, minus the expected loss $(\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})} + \underline{s})(V_1 - V_0)$ from having the follower firm catch up by one step to become level with the leader. Equation (A.2) shows the expected payoff of currently being a follower is equal to the current profit flow π_{-1} plus the expected gains in profit $(\overline{s}^{\alpha-1}\underline{s}^{(1-\alpha_{-1})} + \underline{s})(V_0 - V_{-1})$ from catching up to level with the leader minus the R&D cost $\frac{(\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})})^2}{2}$. In equation (A.3), $\hat{\alpha_0}$ denotes the share of high-skilled workers of the other firm. The Bellman equation shows that in a levelled state, the probability of innovating is lower by $\underline{s}$ as there is no advanced technology that firms can observe. I consider a symmetric Nash equilibrium, in which both firms have the same share of high-skilled workers, that is,

$$\hat{\alpha_0} = \alpha_0.$$

Each firm chooses its worker skill composition to maximise the current expected payoff. This gives the first-order conditions:

$$\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})} = V_0 - V_{-1}, \tag{A.4}$$

$$\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)} = V_1 - V_0. \tag{A.5}$$

Substituting the first-order conditions into the Bellman equations and eliminating the Vs give the following recursive system of reduced form equations:

$$\frac{(\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)})^2}{2} + (r + \underline{s})\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)} - (\pi_1 - \pi_0) = 0, \tag{A.6}$$

$$\frac{(\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})})^2}{2} + \left(r + \underline{s} + \overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)}\right)\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})} - \pi_0 - \frac{(\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)})^2}{2} = 0. \tag{A.7}$$

The solution for α_0 from equation (A.6) is used to solve the equation (A.7). For the case where the rate of time discount $r = 0$ and using the relationship $\pi_0 = (1 - \Delta)\pi_1$, the share of high-skilled workers can be expressed in terms of product market competition, Δ, and the skill level of workers[2]:

$$S_0 = \overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)} = -\underline{s} + \sqrt{\underline{s}^2 + 2\Delta\pi_1} \tag{A.8}$$

$$\alpha_0 = \frac{\ln(\sqrt{\underline{s}^2 + 2\Delta\pi_1} - \underline{s}) - \ln(\underline{s})}{\ln(\overline{s}) - \ln(\underline{s})}, \tag{A.9}$$

$$S_{-1} = \overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})} = -\sqrt{\underline{s}^2 + 2\Delta\pi_1} + \sqrt{\underline{s}^2 + (\sqrt{\underline{s}^2 + 2\Delta\pi_1} - \underline{s})^2 + 2\pi_1} \tag{A.10}$$

$$\alpha_{-1} = \frac{\ln\left(\sqrt{\underline{s}^2 + (\sqrt{\underline{s}^2 + 2\Delta\pi_1} - \underline{s})^2 + 2\pi_1} - \sqrt{\underline{s}^2 + 2\Delta\pi_1}\right) - \ln(\underline{s})}{\ln(\overline{s}) - \ln(\underline{s})}. \tag{A.11}$$

Differentiating equation (A.9) with respect to $\overline{s}$, the equilibrium share of high-skilled workers for a neck-and-neck firm decreases in high-skill endowment:

$$\frac{\partial\alpha_0}{\partial\overline{s}} = -\frac{\alpha_0}{\overline{s}\,[\ln(\overline{s}) - \ln(\underline{s})]} < 0.$$

Similarly, differentiating equation (A.9) with respect to $\underline{s}$, α_0 decreases in low-skill endowment:

$$\frac{\partial\alpha_0}{\partial\underline{s}} = \frac{1}{\underline{s}[\ln(\overline{s}) - \ln(\underline{s})]}\left[\alpha_0 - \frac{\underline{s}}{S_0 + \underline{s}} - 1\right] \in (-1, 0),$$

since[3]

$$\underline{s} < \sqrt{\underline{s}^2 + 2\Delta\pi_1}$$

and $0 \leq \alpha_0 \leq 1$.

For a follower firm, the equilibrium share of high-skilled workers α_{-1} decreases in high-skill endowment $\overline{s}$:

$$\frac{\partial\alpha_{-1}}{\partial\overline{s}} = -\frac{\alpha_{-1}}{\overline{s}\,[\ln(\overline{s}) - \ln(\underline{s})]} < 0.$$

α_{-1} is decreasing in $\underline{s}$ at a slower rate than α_0:

$$\frac{\partial\alpha_{-1}}{\partial\overline{s}} = \frac{1}{\underline{s}[\ln(\overline{s}) - \ln(\underline{s})]}\left\{\alpha_{-1} + \frac{\underline{s}}{S_{-1}}\cdot\right.$$
$$\left.\left[\frac{\underline{s}}{\sqrt{\underline{s}^2 + S_0^2 + 2\pi_1}} - 1 + \left(\frac{S_0}{\sqrt{\underline{s}^2 + S_0^2 + 2\pi_1}} - 1\right)\frac{\partial S_0}{\partial\underline{s}}\right] - 1\right\} \in (-1, 0),$$

since

$$-\frac{\underline{s}}{S_0 + \underline{s}} < \frac{\underline{s}}{S_{-1}}\left[\frac{\underline{s}}{\sqrt{\underline{s}^2 + S_0^2 + 2\pi_1}} - 1 + \left(\frac{S_0}{\sqrt{\underline{s}^2 + S_0^2 + 2\pi_1}} - 1\right)\frac{\partial S_0}{\partial\underline{s}}\right].$$

[2]As α_0 takes values on the interval $[0, 1]$ and $\frac{1}{2} \leq \Delta \leq 1$, equation (A.9) requires that the restriction $\pi_1 \leq \frac{\overline{s}(\overline{s}+2\underline{s})}{2}$ is imposed. This also satisfies the conditions required to avoid undefined values of equation (A.11).

[3]From equation (A.8),

$$\frac{\partial S_0}{\partial\underline{s}} = \frac{\underline{s}}{\sqrt{\underline{s}^2 + 2\Delta\pi_1}} - 1 = -\frac{S_0}{S_0 + \underline{s}} \in (-1, 0).$$

A.2 Proof of Proposition 2

Differentiating equation (A.9) with respect to Δ, one can immediately see that the share of high-skilled workers increases with higher product market competition:

$$\frac{\partial \alpha_0}{\partial \Delta} = \frac{\pi_1}{[\ln(\overline{s}) - \ln(\underline{s})]\, S_0(S_0 + \underline{s})} > 0.$$

In contrast, from equation (A.11), the share of high-skilled workers decreases with higher product market competition for a follower firm:

$$\frac{\partial \alpha_{-1}}{\partial \Delta} = \frac{\pi_1}{[\ln(\overline{s}) - \ln(\underline{s})] S_{-1}(S_0 + \underline{s})} \left[-1 + \left(\frac{S_0}{S_{-1} + S_0 + \underline{s}} \right) \right] < 0,$$

since

$$S_0 < (S_{-1} + S_0 + \underline{s}).$$

A.3 Proof of Proposition 3

Let

$$B = \underline{s}^2 + 2\pi_1.$$

Substituting this into equation (A.10) and expressing as a function of $\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)}$, the equation can be rewritten as

$$\overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})} = \sqrt{(\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)})^2 + B} - \overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)} - \underline{s}.$$

Rewriting the aggregate worker skill composition (4) using the above and the simplified notations S_0 and S_{-1}, the expression can be written as

$$v(S_0) = 4S_0 \frac{\sqrt{S_0^2 + B} - S_0}{\sqrt{S_0^2 + B} + S_0},$$

with

$$v'(S_0) = 4B \left(\frac{1}{\sqrt{S_0^2 + B} + S_0} \right)^2 \left(1 - \frac{2S_0}{\sqrt{S_0^2 + B}} \right).$$

The last term in the above expression

$$f(S_0) = 1 - \frac{2S_0}{\sqrt{S_0^2 + B}}$$

is decreasing in S_0, with a unique value:

$$\tilde{S_0} = \sqrt{\frac{B}{3}},$$

at which $f(S_0) = 0$. Therefore, $v(S_0)$ is quasiconcave with $v'(S_0) \geq 0$ as $S_0 \leq \tilde{S_0}$, and $v'(S_0) \leq 0$ as $S_0 \geq \tilde{S_0}$. The inverted-U pattern occurs whenever $\tilde{S_0} \in (S_0^{\min}, S_0^{\max})$. If $S_0^{\max} \leq \tilde{S_0}$ then the escape-competition will dominate, whereas if $S_0^{\min} \geq \tilde{S}_0$ then the Schumpetarian effect will dominate. To find the range of S_0 values within which the inverted-U shape will occur, let $\eta = \underline{s}/\sqrt{\pi_1}$. The worker skill composition at the maximum and minimum level of competition relative to the unique value of $\tilde{S}_0$ above are

$$\frac{S_0^{\max}}{\tilde{S}_0} = \frac{\sqrt{\eta^2+2}-\eta}{\sqrt{(\eta^2+2)/3}} \quad \text{and} \quad \frac{S_0^{\min}}{\tilde{S}_0} = \frac{\sqrt{\eta+1}-\eta}{\sqrt{(\eta^2+2)/3}}.$$

Therefore, the worker skill composition will attain an inverted-U pattern whenever

$$\sqrt{\eta^2+1} < \sqrt{(\eta^2+2)/3} + \eta < \sqrt{\eta^2+2};$$

The escape-competition effect will strictly dominate over the whole interval $[S_0^{\min}, S_0^{\max}]$ whenever

$$\sqrt{(\eta^2+2)/3} + \eta \geq \sqrt{\eta^2+2};$$

and the Schumpetarian effect will dominate over the whole interval $[S_0^{\min}, S_0^{\max}]$ whenever

$$\sqrt{(\eta^2+2)/3} + \eta < \sqrt{\eta^2+1}.$$

A.4 Proof of Proposition 4

Using equations (3) and (A.10), the steady state probability that the technological state is unlevelled, and therefore the expected technological gap, G, is given by

$$G = \mu_0 \cdot 0 + \mu_1 \cdot 1 = \mu_1 = \frac{2\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)}}{(2\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)} + \overline{s}^{\alpha_{-1}}\underline{s}^{(1-\alpha_{-1})} + \underline{s})}.$$

Using $B = \underline{s}^2 + 2\pi_1$, the expression can be rewritten as

$$\begin{aligned} G &= \frac{2\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)}}{2\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)} + \sqrt{(\overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)})^2 + B}} \\ &= \left[1 + \frac{\sqrt{S_0^2 + B} - S_0}{2S_0}\right]^{-1}. \end{aligned}$$

The technological gap is increasing in $S_0 = \overline{s}^{\alpha_0}\underline{s}^{(1-\alpha_0)}$, the proxy for product market competition. Recall a low-skill driven increase in skill spread refers to a decrease in low-skill endowment, all else constant. I give a proof by assuming the opposite, that is, I show that technological gap is *decreasing* in higher low-skill endowment, therefore, the smaller the low-skill endowment, the wider the technological gap. First, rewrite the above expression:

$$G = \left[1 + \frac{\sqrt{S_0^2 + \underline{s}^2 + 2\pi_1} - S_0}{2S_0}\right]^{-1} = \left[1 + \frac{S_{-1} + \underline{s}}{2S_0}\right]^{-1}$$

Here, G is increasing in $\underline{s}$ because S_0 is decreasing in $\underline{s}$ while $S_{-1} + \underline{s}$ is increasing in $\underline{s}$. The marginal effect of an increase in $\underline{s}$ on S_0 and $S_{-1} + \underline{s}$ are, respectively,

$$\frac{\partial S_0}{\partial \underline{s}} = -\frac{S_0}{S_0 + \underline{s}} \in (-1, 0)$$

The overall worker skill composition for a neck-and-neck firm is decreasing in $\underline{s}$. On the other hand, following from the above:

$$\begin{aligned} \frac{\partial}{\partial \underline{s}} (S_{-1} + \underline{s}) &= \frac{\partial}{\partial \underline{s}} \left(\sqrt{\underline{s}^2 + S_0^2 + 2\pi_1} - S_0 \right) \\ &= \frac{\underline{s}}{\sqrt{\underline{s}^2 + S_0^2 + 2\pi_1}} + \left(\frac{S_0}{\sqrt{\underline{s}^2 + S_0^2 + 2\pi_1}} - 1 \right) \frac{\partial S_0}{\partial \underline{s}} > 0, \end{aligned}$$

since

$$\frac{\partial S_0}{\partial \underline{s}} < 0.$$

This establishes Proposition 4.

A.5 Proof of Proposition 5

Recall

$$B = \underline{s}^2 + 2\pi_1,$$

The marginal effect of $\underline{s}$ on B is

$$\frac{\partial B}{\partial \underline{s}} = 2\underline{s}.$$

The above shows that $\tilde{S}_0 = \sqrt{B/3}$ and $v(\tilde{S}_0)$ are increasing in $\underline{s}$. Suppose $\tilde{S}_0$ is interior to the interval $(S_0^{\min}, S_0^{\max})$. Using the envelope theorem, the marginal effect of $\underline{s}$ on

$$v(\tilde{S}_0) = \max_{S_0 \in (S_0^{\min}, S_0^{\max})} v(S_0)$$

is equal to the direct effect

$$E = \frac{\partial}{\partial B} \left\{ x \frac{\sqrt{x^2 + B} - x}{\sqrt{x^2 + B} + x} \right\},$$

and is positive. The marginal effect of $\underline{s}$ on $v(\tilde{S}_0)$ is $E \cdot (\partial B / \partial \underline{s})$, and is also positive. Therefore, industries with a smaller skill spread (more neck-and-neck industries) have a higher peak in the inverted-U. The peak of the curve occurs when product market competition is such that $S_0 = \sqrt{B/3}$, which is equal to:

$$\begin{aligned} \sqrt{B/3} &= -\underline{s} + \sqrt{\underline{s}^2 + 2\Delta\pi_1} \\ 0 &= -\sqrt{\underline{s}^2 + 2\Delta\pi_1} + \underline{s} + \sqrt{(\underline{s}^{+} 2\pi_2)/3}. \end{aligned} \tag{A.12}$$

If the skill spread affects product market competition negatively, or equivalently, if $d\Delta / d\underline{s} > 0$, then the peak lies farther to the right on the Δ line in more neck-and-neck industries. Here, Δ is implicitly defined by A.12. Applying the implicit function theorem to A.12,

$$\frac{d\Delta}{d\underline{s}} = \frac{\sqrt{\underline{s}^2 + 2\Delta\pi_1}}{\pi_1} \cdot \left(-\frac{\underline{s}}{\sqrt{\underline{s}^2 + 2\Delta\pi_1}} + 1 + \frac{\underline{s}/3}{\sqrt{(\underline{s}^2 + 2\pi_1)/3}} \right) > 0,$$

since

$$\underline{s} < \sqrt{\underline{s}^2 + 2\Delta\pi_1}.$$